# Physics

Exercises on Newton's Laws of Motion in One, Two, and Three Dimensions

# Disclaimer



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# Summary of Exercises

### ***Exercise 1***

Leandro opens his curtains on a Sunday morning and notices that it has started snowing for the first time this year in Trento, Italy. He rushes to wake up his 10-year-old son Aurelio and not much later both are headed ecstatically towards the nearest hill carrying a sleigh of $m_S = 5.20$ kg. Starting from rest at the bottom of the hill, Leandro ($m_L = 76.8$ kg) pulls Aurelio ($m_A = 30.3$ kg) up the hill and when he covered a distance of 12.5 m, he is moving at an instantaneous speed of 2.25 m/s. If Leandro exerts a force of 450 N on the ground (directed along the slope of the hill) due to his pulling activity and knowing that the rope attached to the sleigh is making a $\phi = 10.5°$ angle with the direction in which Leandro is headed, what is the value of the angle of the slope?

### ***Exercise 2***

You're speeding on a 17.5° downhill section of the ring road of Bruges, Belgium, and 80.0 m in front of you the traffic lights suddenly turn orange (after which they switch to red). You hit the breaks and 5.20 s later you come to a halt just in time to avoid crossing the red traffic lights. (1) If the magnitude of the average net force acting on a rear tire of your car during breaking is equal to $F_r = 1,445$ N and knowing that the net force experienced by a front tire is estimated to be 60% higher, what is the mass of your car? (2) At the next lights, you are speeding again, but this time you are traveling on a flat road at a velocity of $\vec{v}_0 = 102 \cdot \vec{i}_x$ km/h and your reaction distance is about 65.0 m. What is the average net force acting on a rear and a front tire?

### ***Exercise 3***

Noora is shopping for her nieces in the mall in Doha, Qatar, and carries a bag ($m_{bag} = 0.150$ kg) containing three presents, which have masses of $m_1 = 0.650$ kg, $m_2 = 2.30$ kg, and $m_3 = 6.45$ kg, respectively. She enters the elevator and goes up one floor, whereby the elevator accelerates at a rate of $a_y = 2.85$ m/s$^2$. (1) If Noora places the bag on the floor next to her left foot, what is the apparent weight of the bag? (2) If she pulls the bag with an acceleration equal to that of the elevator, how does the apparent weight change? (3) If we want the bag to become apparently weightless, with what force $\vec{F}_P$ should Noora pull? (4) If she pulls the bag with a force $\vec{F}_P = 125 \cdot \vec{i}_y$ N, exceeding the force established in part (3), what is the acceleration of the bag?

### ***Exercise 4***

Stina is sitting in the library of the Niels Bohr Institute in Copenhagen, Denmark, studying for her final exam of the course "Mechanics". As she likes to put the theory into practice, she makes a pile of books ($m_b = 3.75$ kg) and applies a force to that stack equal to $\vec{F}_P = 2.3 \cdot \vec{i}_x - 3.9 \cdot \vec{i}_y - 1.8 \cdot \vec{i}_z$ N. If the xz-plane corresponds with the table surface and the y-direction is pointing upwards, and given that the books are at the origin of the coordinate system at $t = 0$ s, (1) what distance and in what direction did the books travel across the table when applying the force for 2.5 s? (2) What is the apparent weight of the stack of books?

## *Exercise 5*

Jakub and Havel just bought a statue ($m_s$ = 120 kg) of the Slovakian artist L'udovít Fulla at a local auction in Ružomberok, Slovakia, and are transporting it to their home on a self-made, 1.50 m-wide bamboo raft ($m_r$ = 12.5 kg) upstream via the Revúca river. The raft is positioned at the center of the 10.0 m-wide river and Jakub and Havel are walking alongside the river on the eastern and western bank, respectively, pulling the raft forward in the southern direction by means of ropes attached to the raft. The x-axis of our coordinate system points in the southern direction, whereby Jakub's rope makes a $\theta_{x,J} = 36.0°$ angle with the x-axis whereas that of Havel forms a $\theta_{x,H} = 48.0°$ angle. As the water level of the river is rather low, their ropes also make an angle of $\theta_{z,J} = 53.0°$ and $\theta_{z,H} = 57.0°$, respectively, with respect to the water surface (the z-axis points upwards).

If Jakub and Havel pull their rope with a force of $F_J = 120$ N and $F_H = 95.0$ N, respectively, and given a $v_{riv} = 1.65$ m/s current at an angle of $\theta_{riv} = 80.0°$ north of east, will the raft hit one of the banks? If so, which one and when? Assume the raft is at rest at $t = 0$ s.

## *Exercise 6*

At a construction site in Lorca, Spain, two heavy pallets stacked with metal pipes ($m_1$ = 490 kg) and plates ($m_2$ = 560 kg) are standing in the scorching sun on top of a 20.0 m tall building and need to be moved into the shadow about 10.0 m to the right. The pallets are connected by cables and one cable is attached to a pulley on the edge of the building, whereby a crate filled with scrap ($m_3$ = 95.0 kg) is hanging at the other end of that cable down the side of the building. The construction site manager Camila is asked to come to the site with her car, which is equipped with a towing hook, and attach a rope to the bottom of the crate of scrap to pull the pallets into the shade.

At the moment Camila starts driving and puts tension on the ropes, the rope between the pulley and the crate of scrap makes an angle of $\theta = 35.0°$ with the vertical whereas an initial angle of $\phi_i = 65.0°$ is formed between the vertical and the rope linking the crate of scrap and Camila's car. (1) Find a general formula for the acceleration of the system "pallets plus crate of scrap". (2) If you know that the crate of scrap hangs 5.00 m below the pulley, how long does Camila need to pull so that the two crates on top of the building move 10.0 m to the right? Assume that the angle $\theta$ remains constant (someone standing on the ground is guiding the crate with a rope), but work with an average for the angle $\phi$ since it increases as Camila drives to the right.

## *Exercise 7*

Tony is attending the course "Experimental Physics" at the University of Auckland, New Zealand, and during one of the lab sessions, he is asked by his supervisor to place a small pulley with a diameter of 5.50 cm between two larger pulleys with a diameter of 10.0 cm (as demonstrated in Fig. 8) and subsequently calculate the acceleration of each of the three masses. Given that a bucket filled with black clay ($m_1 = 5.65$ kg) is hanging from the left pulley, an iron ball ($m_2 = 2.30$ kg) from the middle one, and a stack of three bricks ($m_3 = 4.25$ kg) from the pulley on the right, what values does Tony obtain?

### *Exercise 8*

Harper recently attached a pulley system to the ceiling of her garage in Utah, the United States, and her 7-year-old son Ethan ($m_E = 24.5$ kg) asks his mom whether he can sit in the basket ($m_b = 6.50$ kg) that hangs from the pulley. As Harper knows that the system can easily withstand a weight of 500 N, she reckons it is safe to put her son in the basket. Mischievous as he is, Ethan grabs a stick with a hook on one end and pulls down on the rope that connects the pulley with the metal ring attached to the ceiling. (1) If the rope makes a $\theta = 25.0°$ angle with the horizontal on both sides from the point where the stick touches the rope, with what force $\vec{F}_P$ is Ethan pulling the rope? (2) Given a 2.50 m distance between the pulley and the metal ring, by how much did Ethan manage to pull himself up? (3) Suppose that Ethan increases his pulling force by 35%, which angle does the rope now make with the horizontal? (4) By what distance is Ethan now moving upwards towards the ceiling? Assume for each part of the question that Ethan is hanging still and holds his respective position.

### *Exercise 9*

Robert ($m_R = 66.0$ kg) is asked to play a short intermezzo of $t = 35.0$ s on his professional grand piano ($m_p = 317$ kg) during a festival of classical music in the Brucknerhaus Linz concert hall in Linz, Austria, under some unusual circumstances. Robert will start playing on top of an 18.0 m-long incline, which makes a 23.5° angle with the horizontal, and while he is gradually speeding up the tempo of his musical intermezzo, he is simultaneously being lowered sideways with an ever increasing velocity towards the bottom of the incline. Behind the stage there is an integrated pulley system that has to coordinate Robert's act (see Fig. 12). If the mass of the counterweight B is equal to $m_B = 250$ kg, what should be the mass of counterweight A, so that Robert arrives at the bottom of the incline precisely 35.0 s after he started playing his first note?

### *Exercise 10*

You recently bought a mansion in the outskirts of Brno in the Czech Republic, and, as an architect, you're planning to design a new water fountain to put in the middle of the round square at the end of the driveway that leads to your house. Initially, the water in the fountain gradually flows down via a couple of steps, before falling down vertically. Since your daughter Madlenka just received a large Lego set for her eighth birthday from your brother Petr, you're building a miniature fountain out of Lego bricks, using weights, ropes, and pulleys to model the flow of water. If you would like the water to fall down from the last step at a pace of $a = 6.2$ m/s$^2$ (represented by the acceleration of weight 5 in Fig. 14), under what angle $\theta$ should you build the incline at the end of the last step? Assume the weights have the following masses: $m_1 = 1.5$ kg, $m_2 = 2.5$ kg, $m_3 = 3.5$ kg, $m_4 = 4.5$ kg, $m_5 = 5.5$ kg, and $m_6 = 6.5$ kg.

## ***Exercise 11***

Ashvin ($m_1 = 62.5$ kg) and Opaline ($m_2 = 55.8$ kg) are trying our their brand new wingsuits above the beaches of Port Louis on the Island of Mauritius in preparation of an upcoming skydiving event. Both these daredevils board a separate plane, and Opaline jumps out of her airplane first at a higher altitude than Ashvin. She quickly attains a terminal (i.e., constant) velocity of $v_{0,2} = 27.0$ m/s and descends vertically. At the moment when Opaline reaches the altitude of Ashvin's airplane, which is flying horizontally at $v_{plane} = 65.2$ km/h, Ashvin launches himself from his plane with his arms held close to his body. After a couple of seconds of free fall, Ashvin bumps into Opaline with a velocity of $\vec{v}_{0,1} = 18.1 \cdot \vec{i}_x + 54.0 \cdot \vec{i}_y$ m/s. Right after the collision, Opaline finds herself at the position $\vec{r}_{f,2} = 104 \cdot \vec{i}_x + 170 \cdot \vec{i}_y$ m. If the acceleration vector $\vec{a}$ makes a $\theta = 33.8°$ angle with the vertical, what is the magnitude of the force of impact on Opaline ($\vec{F}_{21}$)?

## ***Exercise 12***

On a casual Wednesday afternoon, Nirmala and Harun are pitching some baseballs on Kemala Beach in Balikpapan, Indonesia. Nirmala is extending her right arm backwards, so that it is positioned 1.00 m above the ground, and launches the baseball ($m_b = 0.15$ kg) with an average force of $F_{throw} = 7.20$ N under an angle of $\theta = 55.0°$ with the horizontal over a distance of 1.50 m in $t_{throw} = 0.25$ s. At the same time when Nirmala is about to release the baseball, a wind of 14.2 kts kicks in at an angle of $\phi = 22.5°$ below the horizontal and generates a corresponding constant force of $F_w = 1.15$ N on the ball. How far backwards should Harun move his left hand—this is the distance $d$ in the same direction as the incoming baseball—when catching the ball 1.50 m above the ground, so that the equivalent mass upon impact is equal to $m_{impact} = 2.35$ kg?

## ***Exercise 13***

Caleb is navigating his Sar 880V Cruiser (with a displacement mass of $m_{dis} = 5.50 \times 10^3$ kg) along the coast of Mayaro Bay, Trinidad and Tobago, with a speed of $v_0 = 13.35$ kts (1 knot =1.852 km/h) heading north towards Ortoire where he will attend a Sunday brunch at his mother's house. One meter to the left of Caleb, two conches that he collected during previous travels are suspended from the same rope, which, in turn, is attached to an aluminum bar. The conch hanging higher is called Lobatus gigas or queen conch ($m_{LG} = 2.50$ kg) and the one below is the Charonia tritonis or giant triton ($m_{CT} = 3.80$ kg). While Caleb is gazing at the Atlantic Ocean through his binoculars, he suddenly spots a blue whale in the northeastern direction at a distance of about 250 m. As he plans to change course and accelerate ($a_i = 0.8226$ m/s$^2$) for 17.8 s until he is 85.0 m away from the whale, Caleb has to take into account an ocean current (southbound) which causes his boat to experience a constant force of $F_{cur} = 1,925$ N. Right at the moment when he sets off in the appropriate direction, the two conches no longer hang vertically but each make a certain angle with the vertical, as shown in Fig. 17. What is the value of these two angles $\alpha$ and $\beta$?

## *Exercise 14*

Zoe has been living in Tresses, France, for the past four years and is now moving to Bordeaux where she is starting a PhD in theoretical physics at the University of Bordeaux. Zoe is almost done packing and she just needs to put one final box ($m_b$) into her car. To avoid overburdening her back, Zoe has placed a ramp in front of her house, so she can slide the moving boxes towards her car. Lying on top of this last box, there is a shelf ($m_s = 2.8$ kg) on which two piles of books ($m_1 = 1.8$ kg and $m_2 = 0.60$ kg, respectively) are placed that she bound together with some rope. When Zoe places the box on the ramp, which makes an angle of $\phi = 18°$ with the ground, the second stack of books falls off the edge of the shelf and is dangling from the rope that is connected to the first pile of books. The rope between the first stack of books and the edge of the shelf now makes an angle of $\gamma = 6.5°$. Since the first pile is now starting to slide towards the edge, Zoe pushes the box down the ramp with a force $\vec{F}_P = 1.3 \times 10^2 \cdot \vec{i}_x$ N in order to keep the first pile on a fixed position on the shelf. What is the mass $m_b$ of the moving box? Assume that the shelf remains in place with respect to the moving box.

## *Exercise 15*

Lixue ($m_L = 55.5$ kg) and Chaun ($m_C = 57.5$ kg) are practicing their trapeze act for the upcoming Lantern Festival in Tianshui (Gansu province), China. At one particular moment during their act, they both jump from opposite sides of the stage from a small platform 9.50 m above ground level onto the aluminum bar of their trapeze ($m_b = 2.10$ kg) and swing towards the middle. As a result, Lixue and Chaun provide their trapeze with an initial push of $\vec{F}_L = 172 \cdot \vec{i}_x$ N and $\vec{F}_C = -188 \cdot \vec{i}_x$ N, respectively. Both trapezes hang from the same height about $x_{trap} = 5.30$ m apart but the cables of Lixue's trapeze are 1.00 m longer ($s_L = 6.00$ m). If you know that both artists can extend their arms for an additional distance of $x_{arm} = 1.00$ m towards each other while swinging on their trapeze, do they manage to touch hands when reaching their farthest point in the horizontal direction? Assume that the origin of the coordinate system is located at the position of the aluminum bar of Lixue's trapeze when her trapeze is hanging vertically and still.

## *Exercise 16*

Ana Laura is a professor at the University of Montevideo, Uruguay, where she teaches the course quantum field theory, and in her spare time Ana Laura loves to build simplified models of planetary surface landers. Today, she is taking one of her latest models for a test flight and all seems to go well. As Ana Laura is guiding her lander ($m_{pl}$) vertically towards the ground, she simultaneously fires the four boosters a first time with a total force of $\vec{F}_1 = 1.3 \times 10^3 \cdot \vec{i}_y$ N, providing the planetary lander with a net upwards acceleration $\vec{a}_1$, so that the lander slows down from $\vec{v}_{0,1} = -8.0 \cdot \vec{i}_y$ m/s to a velocity $\vec{v}$ over a time period $t_1 = 3.6$ s. Immediately afterwards, Ana Laura changes the power supplied by the boosters ($\vec{F}_2$) and after $t_2$ seconds, during which it has been displaced over a distance of $\Delta y_2 = 1.5$ m, the lander has obtained a final velocity of $\vec{v}_{f,2} = 4.4 \cdot \vec{i}_y$ m/s. If the ratio between the acceleration $a_1$ and $a_2$ is equal to 0.442 and given that, due to some technical constraints, the current model cannot accelerate faster than 5.0 m/s$^2$, (1) what is the mass $m_{pl}$ of Ana Laura's planetary lander, and (2) what is the magnitude of $\vec{F}_2$?

## *Exercise 17*

Amadou is writing his Bachelor's thesis at the University of Bamako, Mali, on the mechanics of the Quest Radical compound bow. In particular, Amadou is investigating whether a linear relationship exists between the segment $d$ of the draw length $L$ and the distance $s$ the arrow penetrates into a wooden block after being shot from a certain distance, whereby the mark is positioned at the same height as the bow. After some experimental testing, Amadou finds a relationship $s = \beta \cdot d$ with $\beta = 0.160$. He also know from previous research that a relationship exists between the segment $d$ and the magnitude of the tension force $\vec{T}$, i.e., $d = \gamma \cdot T$ with $\gamma = \frac{1}{450}$. If Amadou shoots an arrow ($m_a = 75.8$ g), which requires $t = 15.0$ ms to leave his bow with a velocity of $v_i = 79.1$ m/s, under an angle of $\phi = 22.0°$, (1) what is the magnitude of the tension force $\vec{T}$ in the string? (2) What angle $\theta$ does the string make with respect to the arrow? (3) What is the magnitude of the force $\vec{F}$ exerted by the string upon the arrow? (4) How deep does the arrow get stuck into the wooden block?

## *Exercise 18*

As the first participants of the cross-country skiing event Skarverennet arrive in Ustaoset, Norway, Kjerstin ($m_K = 68.3$ kg) is enjoying the race from a higher altitude while paragliding above the scene. If we chose the origin of our coordinate system to coincide with Cafe Presttun with the y-axis pointing upwards and the x-axis eastwards, then Kjerstin finds herself at this moment at the position $\vec{r}_0 = 123 \cdot \vec{i}_x + 85.0 \cdot \vec{i}_y + 12.7 \cdot \vec{i}_z$ m with a velocity of $\vec{v}_0 = 5.33 \cdot \vec{i}_x - 2.20 \cdot \vec{i}_y + 0.860 \cdot \vec{i}_z$ m/s. For the next $t_w = 5.50$ s, Kjerstin experiences a wind gust that subjects her to an acceleration of $a = 2.33$ m/s$^2$ and points $\theta_1 = 31.1°$ upwards and $\theta_2 = 68.3°$ north of east. (1) If the gear that Kjerstin is wearing has a mass of $m_g = 5.80$ kg, what is the total force $\vec{F}$ that her seat is exerting upon her during the wind gust? (2) What distance did Kjerstin travel for the duration of the gust? (3) By how much is Kjerstin now farther away from or closer to Cafe Presttun with respect to her initial position?

## *Exercise 19*

María Elena is a Venezuelan artist and she is invited to participate in an exhibition called "Formas y Figuras" ("Forms and Shapes") in the capital Caracas. For this occasion, María Elena selected one of her favourite works, i.e., an intricate piece of art that consists of various blocks made of the wood supplied by the Araguaney tree and carved in the shape of hexagons, nonagons, and dodecagons. The entire complex is held together by an interconnected web of ropes and miniature replications of statues made by other Venezuelan artists, which serve both to render homage to her fellow colleagues and to function as counterweights. María Elena is taking the elevator to her room in Hotel Tamanaco to pick up the last dodecagonal-shaped block and notices that she has gained 15% more weight relative to earlier that morning when she weighed herself in the bathroom—suppose hereby that a scale is installed in the elevator as an extra service for the guests. When riding the elevator back down, she releases for a moment the rope at the right-hand side of the block which causes the miniature statues to slide to the left with an acceleration of $a_{S*} = 0.450$ m/s$^2$ with respect to María Elena. Given a mass of $m_1 = 4.60$ kg, $m_2 = 3.30$ kg, $m_4 = 2.80$ kg, and $m_5 = 3.40$ kg for the other statues and the fact that the outer angle between two consecutive edges of a dodecagon is equal to $\theta = 30.0°$, what is the mass $m_3$ of statue number 3?

## *Exercise 20*

Lovisa ($m_L = 58.6$ kg) is a seasoned rescue professional in the ski area of Åre, Sweden, and with her rescue stretcher ($m_s = 12.2$ kg), which is attached to Lovisa's rescue gear with the help of two metal rods, she just picked up Seo Joon ($m_{SJ} = 85.1$ kg), a South Korean tourist who injured his hip, and is on her way back to the nearest cable car station. Lovisa is standing at the top of a hill and must now gain sufficient speed to reach the station, which is located on top of the next hill. Because she gave away her ski poles to another person in need of rescue, Lovisa is hoping that an initial speed of $v_0 = 4.30$ m/s, gravity, and a constant wind in her back ($F_w = 38.5$ N) on the way down are able to get her to the top of the next hill. Assume that the wind only impacts Lovisa, since Seo Joon is lying close to the ground. (1) If the first slope is $L_1 = 84.0$ m long with an incline of $\phi = 17.8°$ and given that the second hill is 2.00 m higher with a 10.0 m shorter slope and that the wind has turned 180° from the moment she starts moving up the second hill, will Lovisa make it to the cable car station? (2) When Lovisa eventually comes to a halt, what is the tension force in the metal rod? (3) In case that Lovisa does not reach the station, what force should she exert upon her skis in order to accelerate up the hill at $a_u = 1.25$ m/s$^2$?

# Exercise 1

## *Problem Statement*

Leandro opens his curtains on a Sunday morning and notices that it has started snowing for the first time this year in Trento, Italy. He rushes to wake up his 10-year-old son Aurelio and not much later both are headed ecstatically towards the nearest hill carrying a sleigh of $m_S = 5.20$ kg. Starting from rest at the bottom of the hill, Leandro ($m_L = 76.8$ kg) pulls Aurelio ($m_A = 30.3$ kg) up the hill and when he covered a distance of 12.5 m, he is moving at an instantaneous speed of 2.25 m/s. If Leandro exerts a force of 450 N on the ground (directed along the slope of the hill) due to his pulling activity and knowing that the rope attached to the sleigh is making a $\phi = 10.5°$ angle with the direction in which Leandro is headed, what is the value of the angle of the slope?

## *Solution*

Let us in a first instance determine the acceleration of the system through the following equation of motion:

$$v^2 - v_0^2 = 2 \cdot a_x \cdot \Delta x$$

$$\Leftrightarrow \; 2.25^2 - 0^2 = 2 \cdot a_x \cdot 12.5$$

$$\Leftrightarrow \; a_x = \frac{2.25^2}{2 \cdot 12.5} = 0.203 \text{ m/s}^2$$

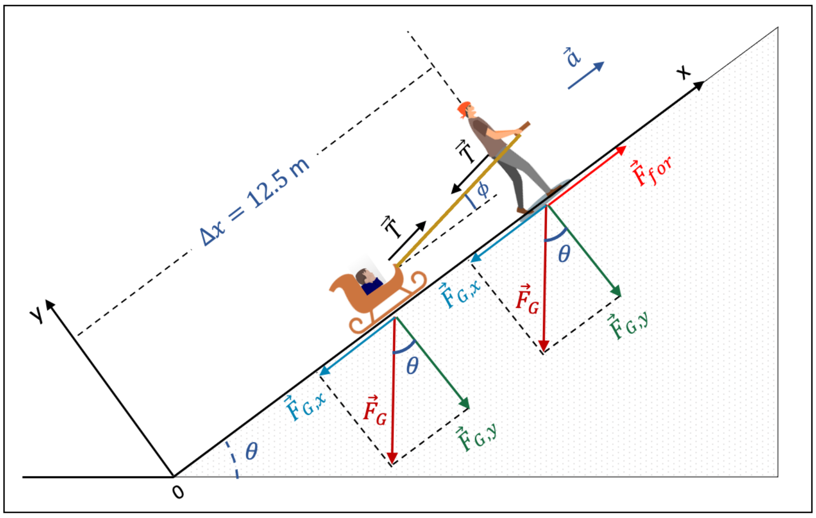


***Figure 1***

When Leandro exerts a force of 450 N on the ground, the ground is returning that force to Leandro, allowing him to move forward ($\vec{F}_{for}$). When applying Newton's second law to Leandro in the x-direction, we obtain an expression for the tension force $\vec{T}$ in the rope, whereby the gravitational force is represented by the vector $\vec{F}_G$:

$$m_L \cdot \vec{a} = \vec{F}_{for} + \vec{T} + \vec{F}_G$$

$$\Leftrightarrow \; m_L \cdot a_x = F_{for} - T \cdot \cos\phi - m_L \cdot g \cdot \sin\theta$$

$$\Leftrightarrow \; T = \frac{F_{for} - m_L \cdot a_x - m_L \cdot g \cdot \sin\theta}{\cos\phi}$$

Considering the forces working on the system "Aurelio plus sleigh" in the x-direction and incorporating the above expression for T, we can find the angle of the hill:

$$(m_A + m_S) \cdot \vec{a} = \vec{T} + \vec{F}_G$$

$$\Leftrightarrow \ (m_A + m_S) \cdot a_x = T \cdot \cos\phi - (m_A + m_S) \cdot (g \cdot \sin\theta)$$

$$\Leftrightarrow \ (m_A + m_S) \cdot a_x = \left[\frac{F_{for} - m_L \cdot a_x - m_L \cdot g \cdot \sin\theta}{\cos\phi}\right] \cdot \cos\phi - (m_A + m_S) \cdot (g \cdot \sin\theta)$$

$$\Leftrightarrow \ \sin\theta = \frac{F_{for} - (m_L + m_A + m_S) \cdot a_x}{(m_L + m_A + m_S) \cdot g}$$

$$\Leftrightarrow \ \sin\theta = \frac{450 - (76.8 + 30.3 + 5.20) \cdot 0.203}{(76.8 + 30.3 + 5.20) \cdot 9.81} = 0.388$$

$$\Leftrightarrow \ \theta = 22.8^\circ$$

As a check, let us verify that the forward force $\vec{F}_{for}$ must indeed be larger than the sum of $\vec{T}$ and $\vec{F}_G$ working on Leandro to ensure that he is able to accelerate in the forward direction:

$$F_{for} > T \cdot \cos\phi + m_L \cdot g \cdot \sin\theta$$

$$\Leftrightarrow \ F_{for} > \left[\frac{F_{for} - m_L \cdot a_x - m_L \cdot g \cdot \sin\theta}{\cos\phi}\right] \cdot \cos\phi + m_L \cdot g \cdot \sin\theta$$

$$\Leftrightarrow \ 450 > \left[\frac{450 - 76.8 \cdot 0.203 - 76.8 \cdot 9.81 \cdot \sin(22.8^\circ)}{\cos(10.5^\circ)}\right] \cdot \cos(10.5^\circ) + 76.8 \cdot 9.81 \cdot \sin(22.8^\circ)$$

$$\Leftrightarrow \ 450 > 145 \cdot \cos(10.5^\circ) + 76.8 \cdot 9.81 \cdot \sin(22.8^\circ)$$

$$\Leftrightarrow \ 450 > 142 + 292$$

$$\Leftrightarrow \ 450 > 434$$

The difference between $\vec{F}_{for}$ and $(\vec{T}+\vec{F}_G)$ is equal to the net force $\vec{F}_{net} = \vec{F}_{for} - (\vec{T}+\vec{F}_G) = (450-434) \cdot \vec{i}_x$ N $= 15.6 \cdot \vec{i}_x$ N providing Leandro with an acceleration. We can verify that $m_L = \frac{F_{net}}{a_x} = \frac{15.6}{0.203} = 76.8$ kg.

# Exercise 2

## ***Problem Statement***

You're speeding on a 17.5° downhill section of the ring road of Bruges, Belgium, and 80.0 m in front of you the traffic lights suddenly turn orange (after which they switch to red). You hit the breaks and 5.20 s later you come to a halt just in time to avoid crossing the red traffic lights. (1) If the magnitude of the average net force acting on a rear tire of your car during breaking is equal to $F_r = 1,445$ N and knowing that the net force experienced by a front tire is estimated to be 60% higher, what is the mass of your car? (2) At the next lights, you are speeding again, but this time you are traveling on a flat road at a velocity of $\vec{v}_0 = 102 \cdot \vec{i}_x$ km/h and your reaction distance is about 65.0 m. What is the average net force acting on a rear and a front tire?

## ***Solution***

(1) We calculate the magnitude of the deceleration $\vec{a}_x$ of your car by first finding an expression for the initial speed $v_0$ and incorporating that into a second equation of motion (whereby the final speed $v$ of your car is equal to $v = 0$ m/s):

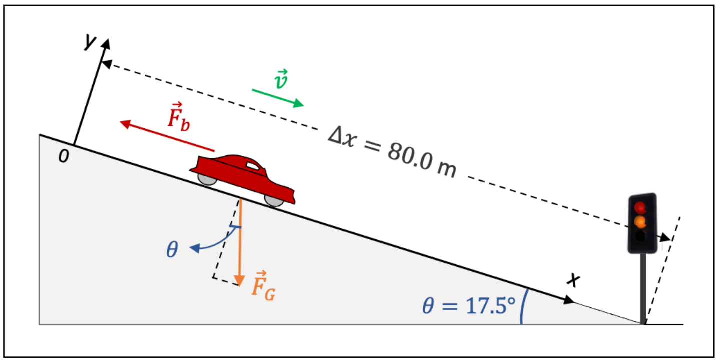


***Figure 2***

$$\begin{cases} v = v_0 + a_x \cdot t \\ \Leftrightarrow\ v_0 = v - a_x \cdot t \\ \\ \Delta x = v_0 \cdot t + \dfrac{a_x}{2} \cdot t^2 \\ \Leftrightarrow\ \Delta x = (v - a_x \cdot t) \cdot t + \dfrac{a_x}{2} \cdot t^2 = v \cdot t - \dfrac{a_x \cdot t^2}{2} \\ \Leftrightarrow\ a_x = (v \cdot t - \Delta x) \cdot \dfrac{2}{t^2} = (0 \cdot 5.20 - 80.0) \cdot \dfrac{2}{5.20^2} = -5.92 \text{ m/s}^2 \end{cases}$$

The mass of your car is determined by recurring to Newton's second law. In a first instance, let us determine the total average net force $\vec{F}_b$ acting on your car while breaking (note that this force points into the negative x-direction):

$$\vec{F}_b = 2 \cdot \vec{F}_r + 2 \cdot \left(1.6 \cdot \vec{F}_r\right) = -2 \cdot 1,445 \cdot \vec{i}_x - 2 \cdot \left(1.6 \cdot 1,445 \cdot \vec{i}_x\right) = -7,520 \cdot \vec{i}_x \text{ N}$$

Applying Newton's second law to your car in the x-direction, we obtain the following mass $m_{car}$:

$$\vec{F}_b + \vec{F}_G = m_{car} \cdot \vec{a}_x \quad \Leftrightarrow \quad -F_b \cdot \vec{i}_x + m_{car} \cdot g \cdot \sin\theta \cdot \vec{i}_x = -m_{car} \cdot a_x \cdot \vec{i}_x$$

$$\Leftrightarrow \quad m_{car} = \frac{F_b}{a_x + g \cdot \sin\theta}$$

$$= \frac{7,520}{5.92 + 9.81 \cdot \sin(17.5°)}$$

$$= 847 \text{ kg}$$

(2) First, we calculate the magnitude of the deceleration $\vec{a}$ of your car:

$$v^2 - v_0^2 = 2 \cdot a \cdot \Delta x \quad \Leftrightarrow \quad a = \frac{v^2 - v_0^2}{2 \cdot \Delta x} = \frac{0^2 - 28.3^2}{2 \cdot 65.0} = -6.18 \text{ m/s}^2$$

The net force $\vec{F}_{net}$ acting upon your car as a whole while breaking is equal to:

$$\vec{F}_{net} = m_{car} \cdot \vec{a} = -847 \cdot 6.18 = -5,230 \cdot \vec{i}_x \text{ N}$$

Since the magnitude of the average net force $\vec{F}_f$ experienced by a front tire is 1.6 times that of a rear tire, we find the magnitude of the average net force $\vec{F}_r$ acting upon a rear tire as follows:

$$F_{net} = 2 \cdot F_r + 2 \cdot F_f = 2 \cdot F_r + 2 \cdot (1.6 \cdot F_r) = 5.2 \cdot F_r \quad \Leftrightarrow \quad F_r = \frac{F_{net}}{5.2} = \frac{5,230}{5.2} = 1,000 \text{ N}$$

The magnitude of the average net force $\vec{F}_f$ acting on a front tire is then equal to $F_f = 1.6 \cdot F_r = 1.6 \cdot 1,000 = 1,600$ N.

# Exercise 3

## *Problem Statement*

Noora is shopping for her nieces in the mall in Doha, Qatar, and carries a bag ($m_{bag} = 0.150$ kg) containing three presents, which have masses of $m_1 = 0.650$ kg, $m_2 = 2.30$ kg, and $m_3 = 6.45$ kg, respectively. She enters the elevator and goes up one floor, whereby the elevator accelerates at a rate of $a_y = 2.85$ m/s$^2$. (1) If Noora places the bag on the floor next to her left foot, what is the apparent weight of the bag? (2) If she pulls the bag with an acceleration equal to that of the elevator, how does the apparent weight change? (3) If we want the bag to become apparently weightless, with what force $\vec{F}_P$ should Noora pull? (4) If she pulls the bag with a force $\vec{F}_P = 125 \cdot \vec{i}_y$ N, exceeding the force established in part (3), what is the acceleration of the bag?

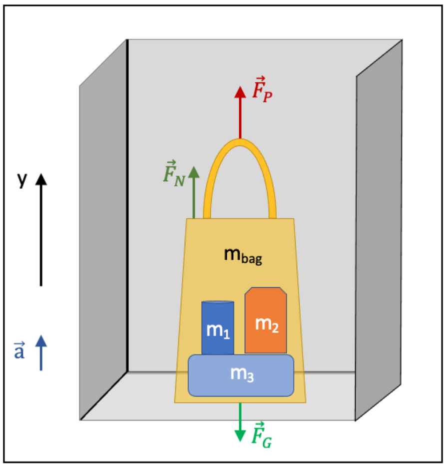


***Figure 3***

## *Solution*

(1) When considering the system "bag with three presents" as a whole, let us define the total mass of the system $m_{tot}$ in the following way:

$$m_{tot} = m_{bag} + m_1 + m_2 + m_3 = 0.150 + 0.650 + 2.30 + 6.45 = 9.55 \text{ kg}$$

We can find the apparent weight, which is represented by the normal force $\vec{F}_N$, as follows:

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_G$$

$$\Leftrightarrow \; m_{tot} \cdot a_y = F_N - m_{tot} \cdot g$$

$$\Leftrightarrow \; F_N = m_{tot} \cdot (g + a_y)$$

$$= 9.55 \cdot (9.81 + 2.85)$$

$$= 121 \text{ N}$$

(2) Pulling up the bag with an acceleration equal to that of the elevator means that the force $\vec{F}_P$ is equal to the net force $\vec{F}_{net}$. In this case, the apparent weight of the bag becomes:

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_G + \vec{F}_P$$

$$\Leftrightarrow\ m_{tot} \cdot a_y = F_N - m_{tot} \cdot g + m_{tot} \cdot a_y$$

$$\Leftrightarrow\ F_N = m_{tot} \cdot g$$

$$= 9.55 \cdot 9.81$$

$$= 93.7 \text{ N}$$

(3) To render the bag apparently weightless, Noora should apply the condition $\vec{F}_N = 0$ N. We can then write:

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_G + \vec{F}_P \quad \Leftrightarrow\ m_{tot} \cdot a_y = F_N - m_{tot} \cdot g + F_P$$

$$\Leftrightarrow\ F_P = m_{tot} \cdot (g + a_y) - F_N$$

$$= 9.55 \cdot (9.81 + 2.85) - 0.00$$

$$= 121 \text{ N}$$

This is the same answer as part (1), which makes sense since the pulling force is now performing, in some way, the job of the normal force in (1).

(4) Pulling the bag with a force greater than $F_P = 121$ N means that Noora will now be effectively lifting the bag up. As a result, the bag is no longer experiencing a normal force by the elevator since the physical contact disappears. If Noora pulls the bag with a force equal to $\vec{F}_P = 125\vec{i}_y$ N, the acceleration of the bag is found as follows:

$$\vec{F}_{net} = \vec{F}_G + \vec{F}_P$$

$$\Leftrightarrow\ m_{tot} \cdot a = -m_{tot} \cdot g + F_P$$

$$\Leftrightarrow\ a = -g + \frac{F_P}{m_{tot}}$$

$$= -9.81 + \frac{125}{9.55}$$

$$= 3.28 \text{ m/s}^2$$

Keep in mind that this acceleration is viewed by someone positioned in a stationary framework. From the perspective of Noora inside the elevator, the acceleration becomes $a = 3.28 - 2.85 = 0.429$ m/s$^2$ or $a = \frac{F_{net}}{m_{tot}} = \frac{125-121}{9.55} = 0.429$ m/s$^2$ (remember to use not rounded figures during intermediate calculations).

# Exercise 4

## ***Problem Statement***

Stina is sitting in the library of the Niels Bohr Institute in Copenhagen, Denmark, studying for her final exam of the course "Mechanics". As she likes to put the theory into practice, she makes a pile of books ($m_b = 3.75$ kg) and applies a force to that stack equal to $\vec{F}_P = 2.3 \cdot \vec{i}_x - 3.9 \cdot \vec{i}_y - 1.8 \cdot \vec{i}_z$ N. If the xz-plane corresponds with the table surface and the y-direction is pointing upwards, and given that the books are at the origin of the coordinate system at $t = 0$ s, (1) what distance and in what direction did the books travel across the table when applying the force for 2.5 s? (2) What is the apparent weight of the stack of books?

## ***Solution***

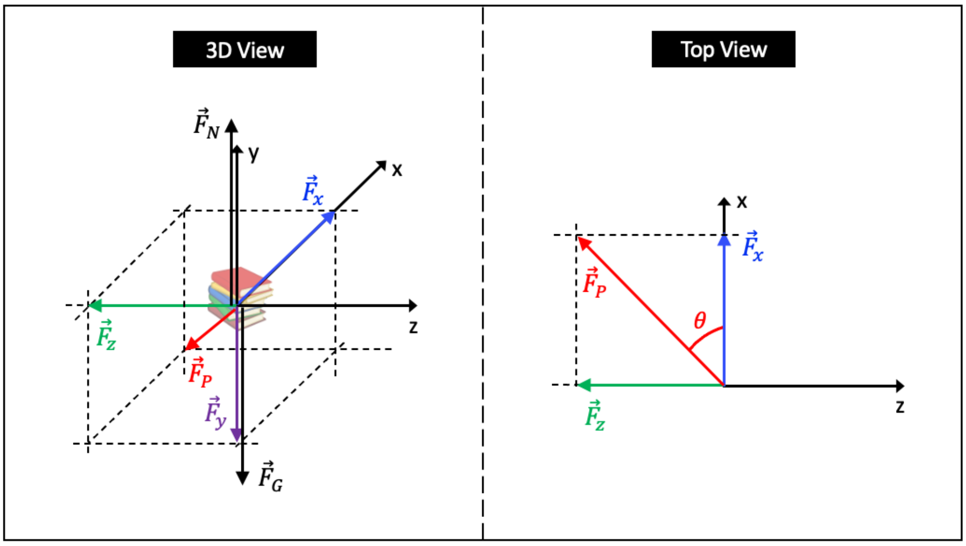


***Figure 4***

(1) The acceleration in the x- and z-direction as well as the x- and z-components of the position vector of the pile of books are calculated as follows:

$$\begin{cases} F_x = m_b \cdot a_x \\ \Leftrightarrow a_x = \dfrac{F_x}{m_b} = \dfrac{2.3}{3.75} = 0.613 \text{ m/s}^2 \end{cases} \qquad \begin{aligned} x &= \frac{a_x}{2} \cdot t^2 \\ &= \frac{0.613}{2} \cdot 2.5^2 = 1.92 \text{ m} \end{aligned}$$

$$\begin{cases} F_z = m_b \cdot a_z \\ \Leftrightarrow a_z = \dfrac{F_z}{m_b} = \dfrac{-1.8}{3.75} = -0.480 \text{ m/s}^2 \end{cases} \qquad \begin{aligned} z &= \frac{a_z}{2} \cdot t^2 \\ &= \frac{(-0.48)}{2} \cdot 2.5^2 = -1.50 \text{ m} \end{aligned}$$

The distance Stina slid the books across the table becomes:

$$d = \sqrt{x^2 + z^2} = \sqrt{1.92^2 + (-1.50)^2} = 2.4 \text{ m}$$

The pile of books moved in the xz-plane under the following angle with respect to the x-axis:

$$\theta = \tan^{-1}\left(\frac{z}{x}\right) = \tan^{-1}\left(\frac{1.50}{1.92}\right) = 38^\circ$$

(2) To find the apparent weight of the stack of books, we need to focus on the y-direction:

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_G + \vec{F}_P$$

$$\Leftrightarrow \; 0 = F_N - m_b \cdot g - F_y$$

$$\Leftrightarrow \; F_N = m_b \cdot g + F_y$$

$$= 3.75 \cdot 9.81 + 3.9$$

$$= 41 \text{ N}$$

This is greater compared to the actual weight of the books $F_G = m_b \cdot g = 3.75 \cdot 9.81 = 37$ N due to the applied force in the negative y-direction.

# Exercise 5

## ***Problem Statement***

Jakub and Havel just bought a statue ($m_s$ = 120 kg) of the Slovakian artist L'udovít Fulla at a local auction in Ružomberok, Slovakia, and are transporting it to their home on a self-made, 1.50 m-wide bamboo raft ($m_r$ = 12.5 kg) upstream via the Revúca river. The raft is positioned at the center of the 10.0 m-wide river and Jakub and Havel are walking alongside the river on the eastern and western bank, respectively, pulling the raft forward in the southern direction by means of ropes attached to the raft. The x-axis of our coordinate system points in the southern direction, whereby Jakub's rope makes a $\theta_{x,J} = 36.0°$ angle with the x-axis whereas that of Havel forms a $\theta_{x,H} = 48.0°$ angle. As the water level of the river is rather low, their ropes also make an angle of $\theta_{z,J} = 53.0°$ and $\theta_{z,H} = 57.0°$, respectively, with respect to the water surface (the z-axis points upwards).

If Jakub and Havel pull their rope with a force of $F_J$ = 120 N and $F_H$ = 95.0 N, respectively, and given a $v_{riv}$ = 1.65 m/s current at an angle of $\theta_{riv} = 80.0°$ north of east, will the raft hit one of the banks? If so, which one and when? Assume the raft is at rest at $t$ = 0 s.

## ***Solution***

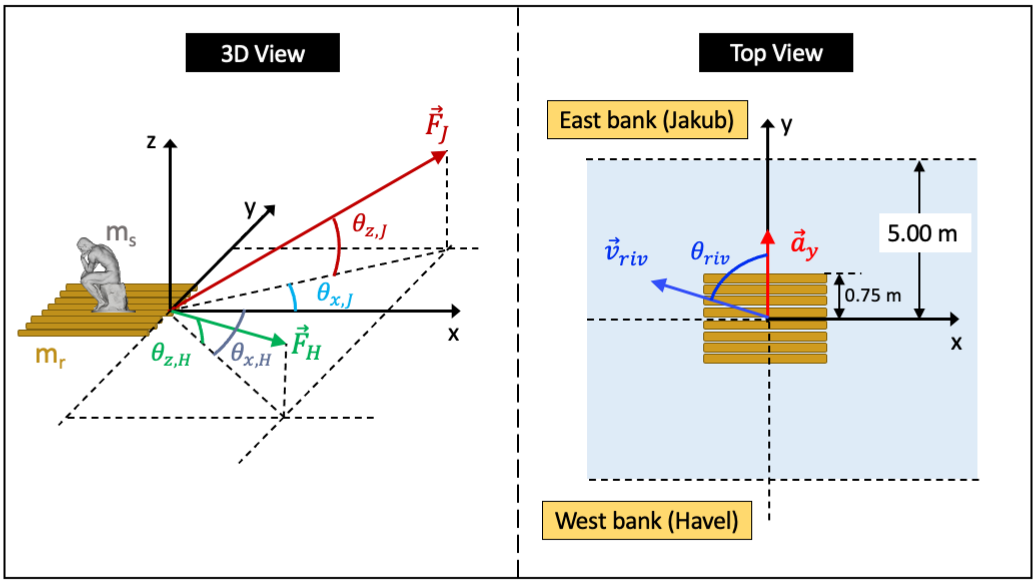


***Figure 5***

As the raft moves in the xy-plane of our coordinate system, let us start with calculating the magnitude of the x- and y-components of the resultant force vector $\vec{F}_R$, i.e., $F_x$ and $F_y$, respectively:

$$\begin{cases} F_x &= (F_J \cdot \cos\theta_{z,J}) \cdot \cos\theta_{x,J} + (F_H \cdot \cos\theta_{z,H}) \cdot \cos\theta_{x,H} \\ &= [120 \cdot \cos(53.0°)] \cdot \cos(36.0°) + [95.0 \cdot \cos(57.0°)] \cdot \cos(48.0°) \\ &= 93.0 \text{ N} \\ \\ F_y &= (F_J \cdot \cos\theta_{z,J}) \cdot \sin\theta_{x,J} + (F_H \cdot \cos\theta_{z,H}) \cdot \sin\theta_{x,H} \\ &= [120 \cdot \cos(53.0°)] \cdot \sin(36.0°) - [95.0 \cdot \cos(57.0°)] \cdot \sin(48.0°) \\ &= 4.00 \text{ N} \end{cases}$$

Given a positive net force component in the y-direction, we know that the raft is being accelerated in the positive y-direction, i.e., towards the east bank where Jakub is walking. The corresponding magnitude of the acceleration $\vec{a}_y$ is equal to:

$$a_y = \frac{F_y}{m_s + m_r} = \frac{4.00}{120 + 12.5} = 3.02 \times 10^{-2} \text{ m/s}^2$$

In other words, if both Jakub and Havel maintain their current pulling force, the raft will eventually hit the east bank. Since the 1.50 m-wide raft started out in the middle of the 10.0 m-wide river, the east side of the raft was initially 4.25 m away from the east bank. In order to calculate when it is going to hit the east bank we need to solve the following equation of motion, taking into account the current of $v_{riv} = 1.65$ m/s at an angle of 80.0° north of east:

$$y(t) = y_0 + v_{riv,y} \cdot t + \frac{a_y}{2} \cdot t^2$$

$$\Leftrightarrow\ y(t) = y_0 + [v_{riv} \cdot \cos\theta_{riv}] \cdot t + \frac{a_y}{2} \cdot t^2$$

$$\Leftrightarrow\ 5.00 = 0.75 + [1.65 \cdot \cos(80.0°)] \cdot t + \frac{0.0302}{2} \cdot t^2$$

$$\Leftrightarrow\ 0 = -4.25 + [1.65 \cdot \cos(80.0°)] \cdot t + \frac{0.0302}{2} \cdot t^2$$

The physically sensible solution (whereby $t \geq 0$ s) that we obtain by solving the above quadratic equation is equal to $t = 9.79$ s. To avoid hitting the east bank within the first 9.79 s of transporting the Fulla statue on their bamboo raft, Jakub could, for instance, pull with a lesser force or Havel could increase his pulling force.

# Exercise 6

## *Problem Statement*

At a construction site in Lorca, Spain, two heavy pallets stacked with metal pipes ($m_1 = 490$ kg) and plates ($m_2 = 560$ kg) are standing in the scorching sun on top of a 20.0 m tall building and need to be moved into the shadow about 10.0 m to the right. The pallets are connected by cables and one cable is attached to a pulley on the edge of the building, whereby a crate filled with scrap ($m_3 = 95.0$ kg) is hanging at the other end of that cable down the side of the building. The construction site manager Camila is asked to come to the site with her car, which is equipped with a towing hook, and attach a rope to the bottom of the crate of scrap to pull the pallets into the shade.

At the moment Camila starts driving and puts tension on the ropes, the rope between the pulley and the crate of scrap makes an angle of $\theta = 35.0°$ with the vertical whereas an initial angle of $\phi_i = 65.0°$ is formed between the vertical and the rope linking the crate of scrap and Camila's car. (1) Find a general formula for the acceleration of the system "pallets plus crate of scrap". (2) If you know that the crate of scrap hangs 5.00 m below the pulley, how long does Camila need to pull so that the two crates on top of the building move 10.0 m to the right? Assume that the angle $\theta$ remains constant (someone standing on the ground is guiding the crate with a rope), but work with an average for the angle $\phi$ since it increases as Camila drives to the right.

## *Solution*

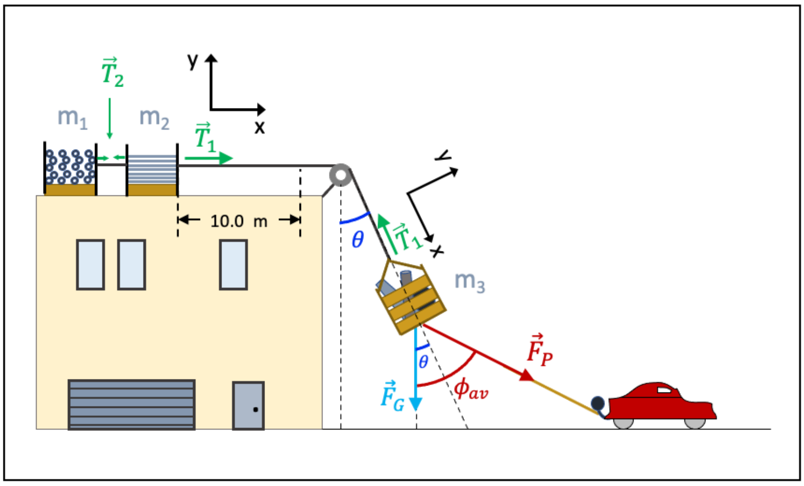


***Figure 6***

(1) Applying Newton's second law to the three masses gives the following three equations for the x-direction:

$$\begin{cases} \vec{F}_{net,1} = \vec{T}_2 & \vec{F}_{net,2} = \vec{T}_1 + \vec{T}_2 & \vec{F}_{net,3} = \vec{T}_1 + \vec{F}_G + \vec{F}_P \\ \Leftrightarrow m_1 \cdot a = T_2 & \Leftrightarrow m_2 \cdot a = T_1 - T_2 & \Leftrightarrow m_3 \cdot a = -T_1 + m_3 \cdot g \cdot \cos\theta + \\ & & \quad F_P \cdot \cos(\phi_{av} - \theta) \end{cases}$$

Applying Newton's second law to the crate of scrap in the y-direction gives an expression for the magnitude of the pulling force $\vec{F}_P$:

$$\vec{F}_{net,3} = \vec{F}_P + \vec{F}_G = \vec{0} \quad \Leftrightarrow \quad F_P \cdot \sin(\phi_{av} - \theta) - m_3 \cdot g \cdot \sin\theta = 0 \quad \Leftrightarrow \quad F_P = m_3 \cdot g \cdot \frac{\sin\theta}{\sin(\phi_{av} - \theta)}$$

Replacing $T_2$ in the equation of the second mass by the expression for $T_2$ obtained from the equation of the first mass, allows us to write $T_1$ as $T_1 = (m_1 + m_2) \cdot a$. If we insert the expressions for $T_1$ and $F_P$ into the equation of the third mass (with respect to the x-direction), we find the general equation for the acceleration (whereby we make use of the angle addition theorem "$\sin(A + B) = \sin A \cos B + \cos A \sin B$"):

$$m_3 \cdot a = -T_1 + m_3 \cdot g \cdot \cos\theta + F_P \cdot \cos(\phi_{av} - \theta)$$

$$\Leftrightarrow \; m_3 \cdot a = -[(m_1 + m_2) \cdot a] + m_3 \cdot g \cdot \cos\theta + \left[m_3 \cdot g \cdot \frac{\sin\theta}{\sin(\phi_{av} - \theta)}\right] \cdot \cos(\phi_{av} - \theta)$$

$$\Leftrightarrow \; (m_1 + m_2 + m_3) \cdot a = \frac{m_3 \cdot g}{\sin(\phi_{av} - \theta)} \cdot [\sin(\phi_{av} - \theta) \cdot \cos\theta + \sin\theta \cdot \cos(\phi_{av} - \theta)]$$

$$\Leftrightarrow \; a = \frac{m_3 \cdot g}{m_1 + m_2 + m_3} \cdot \frac{\sin\phi_{av}}{\sin(\phi_{av} - \theta)}$$

(2) Since the vertical distance between the pulley and the crate of scrap measures $h = 5.00$ m and the rope between the pulley and the crate of scrap makes a $\theta = 35.0°$ angle with the vertical, we can determine the length $s$ of that part of the rope as well as the horizontal distance $x$ between the building and the crate of scrap:

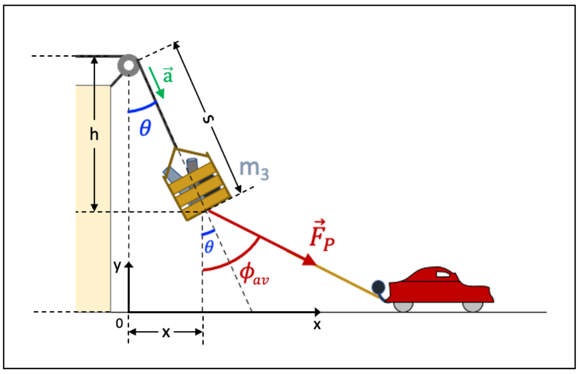


***Figure 7***

$$\begin{cases} s = \dfrac{h}{\cos\theta} & x = h \cdot \tan\theta \\ \quad = \dfrac{5.00}{\cos(35.0°)} & \quad = 5.00 \cdot \tan(35.0°) \\ \quad = 6.10 \text{ m} & \quad = 3.50 \text{ m} \end{cases}$$

When Camila drives her car forwards, it is the rope that needs to be displaced by 10.0 m. That is, the length $s$ must increase by 10.0 m from $s = 6.10$ m to $s' = 16.1$ m. Therefore, the new horizontal distance $x'$ between the building and the crate becomes $x' = s' \cdot \sin\theta = 16.1 \cdot \sin(35.0°) = 9.24$ m. This means that the crate moves to the right over a distance $d_{x1}$ equal to $d_{x1} = x' - x = 9.24 - 3.50 = 5.74$m.

Similarly, the new distance $h'$ between the pulley and the crate becomes $h' = s' \cdot \cos\theta = 16.1 \cdot \cos(35.0°) = 13.2$ m.

Let us now calculate the final angle $\phi_f$ and the average angle $\phi_{av}$. Given that the length $L$ of the rope between the crate and the car is equal to $L = \frac{20.0-h}{\cos\phi_i} = \frac{20.0-5.00}{\cos(65.0°)} = 35.5$ m, the final angle $\phi_f$ then becomes $\phi_f = \cos^{-1}\left(\frac{20.0-h'}{L}\right) = \cos^{-1}\left(\frac{20.0-13.2}{35.5}\right) = 78.9°$. Therefore, the value of the average angle $\phi_{av}$ is equal to $\phi_{av} = \frac{1}{2} \cdot (\phi_i + \phi_f) = \frac{1}{2} \cdot (65.0° + 78.9°) = 72.0°$.

As the angle $\phi$ grows larger, the horizontal distance $d_{x2}$ between the crate and the car also slightly increases and can be calculated as follows:

$$d_{x2} = L \cdot \sin\phi_f - L \cdot \sin\phi_i = 35.5 \cdot \sin(78.9°) - 35.5 \cdot \sin(65.0°) = 34.8 - 32.2 = 2.67 \text{ m}$$

The total displacement $\Delta x$ of Camila's car is then equal to $\Delta x = d_{x1} + d_{x2} = 5.74 + 2.67 = 8.40$ m. Since we worked with an average angle $\phi_{av}$, we can assume that the magnitude of the acceleration $\vec{a}_x$ with which Camila is able to move to the right is equal to the x-component of the acceleration of the system "pallets plus crate of scrap". In other words:

$$\begin{aligned} a_x = a \cdot \sin\theta &= \left[\frac{m_3 \cdot g}{m_1 + m_2 + m_3} \cdot \frac{\sin\phi_{av}}{\sin(\phi_{av} - \theta)}\right] \cdot \sin\theta \\ &= \left[\frac{95.0 \cdot 9.81}{490 + 560 + 95.0} \cdot \frac{\sin(72.0°)}{\sin(72.0° - 35.0°)}\right] \cdot \sin(35.0°) \\ &= 0.738 \text{ m/s}^2 \end{aligned}$$

We can now write the equation of motion for Camila's car in the x-direction and calculate the time needed for Camila to slide the two pallets on top of the building 10.0 m to the right into the shade:

$$\begin{aligned} \Delta x(t) = v_{0x} \cdot t + \frac{a_x}{2} \cdot t^2 \quad &\Leftrightarrow \quad \Delta x = 0 \cdot t + \frac{a_x}{2} \cdot t^2 \\ &\Leftrightarrow \quad 8.40 = \frac{0.738}{2} \cdot t^2 \\ &\Leftrightarrow \quad t = 4.77 \text{ s} \end{aligned}$$

Note that in this exercise we ignored any friction between the pallets and the surface of the rooftop—for exercises that include friction see the exercise package "Applications of Newton's Laws"—which means that merely due to the gravitational downward pull of the crate of scrap the two pallets start moving right away. In a sense, this makes the presence of Camila's car redundant. However, the role of Camila's car here is to make the crate of scrap move downwards under an angle $\theta$.

# Exercise 7

## *Problem Statement*

Tony is attending the course "Experimental Physics" at the University of Auckland, New Zealand, and during one of the lab sessions, he is asked by his supervisor to place a small pulley with a diameter of 5.50 cm between two larger pulleys with a diameter of 10.0 cm (as demonstrated in Fig. 8) and subsequently calculate the acceleration of each of the three masses. Given that a bucket filled with black clay ($m_1$ = 5.65 kg) is hanging from the left pulley, an iron ball ($m_2$ = 2.30 kg) from the middle one, and a stack of three bricks ($m_3$ = 4.25 kg) from the pulley on the right, what values does Tony obtain?

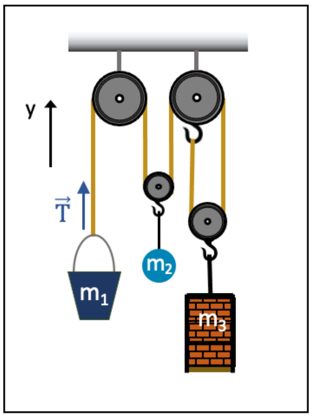


***Figure 8***

## *Solution*

Before writing the equation of motion for the three masses, we have to determine the tension force in the cable connecting the iron ball and the pile of bricks to their respective pulley. In the case of the iron ball (the reasoning applies equally to the stack of bricks), Fig. 9 shows that this tension force is twice the value of the tension force $\vec{T}$ running through the rope at both sides of the pulley—the reason why the tension force in the rope at both sides of the pulley is identical is because we are implicitly accepting an ideal pulley system, i.e., we make abstraction of any rotational friction present and assume that the mass of both the pulley and the rope is negligible. As the forces within the pulley itself must balance each other out, the two tension forces $\vec{T}$ pointing upwards must equal the force pointing downwards ($2\vec{T}$).

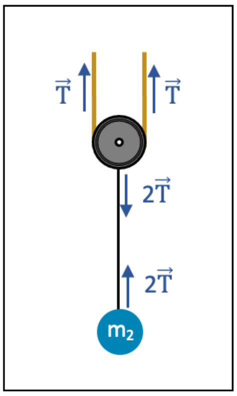


***Figure 9***

We can now write the equation of motion for the bucket, the iron ball, and the pile of bricks, respectively:

$$\begin{cases} \text{Bucket:} & T - m_1 \cdot g = m_1 \cdot a_1 \\ \text{Iron ball:} & 2 \cdot T - m_2 \cdot g = m_2 \cdot a_2 \\ \text{Bricks:} & 2 \cdot T - m_3 \cdot g = m_3 \cdot a_3 \end{cases}$$

In the above three equations, we have four unknown variables, which means that we need another constraint in order to determine the value of these variables. Looking closely at Fig. 10, we can see that the length L of the rope remains constant, regardless of the motion of the three masses. In other words, although the vertical distances $y_1$, $y_2$, and $y_3$ may vary, the rope's length does not.

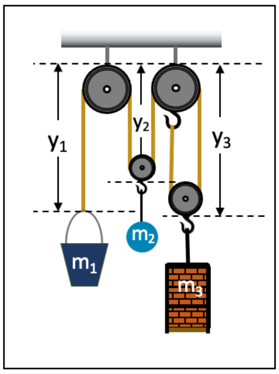


***Figure 10***

Translating this constraint into mathematical language gives us the following equation:

$$y_1 + 2 \cdot y_2 + y_3 + (y_3 - 0.100) = L$$

Taking the second derivative of the distance with respect to time gives an expression for the acceleration so that taking the second derivative of the above equation results in a fourth constraint:

$$a_1 + 2 \cdot a_2 + 2 \cdot a_3 = 0$$

We can now calculate the acceleration of the three masses. Replacing "$2 \cdot T$" in the third equation of motion by the expression for "$2 \cdot T$" obtained from the second equation of motion, we can write the third equation of motion as follows:

$$(m_2 \cdot a_2 + m_2 \cdot g) - m_3 \cdot g = m_3 \cdot a_3$$

Replacing T in the second equation of motion by the expression for T obtained from the first equation of motion and simultaneously replacing $a_1$ by the expression for $a_1$ obtained from the fourth constraint, we can write the second equation of motion in the following way:

$$2 \cdot (m_1 \cdot g + m_1 \cdot [-2 \cdot a_2 - 2 \cdot a_3]) - m_2 \cdot g = m_2 \cdot a_2$$

$$\Leftrightarrow \ (2 \cdot m_1 - m_2) \cdot g = (4 \cdot m_1 + m_2) \cdot a_2 + 4 \cdot m_1 \cdot a_3$$

Combining the previous two equations provides us with an expression for the acceleration $a_2$ of the iron ball by replacing $a_3$ in the last equation by the expression for $a_3$ obtained from the first of the last two equations:

$$(2 \cdot m_1 - m_2) \cdot g = (4 \cdot m_1 + m_2) \cdot a_2 + 4 \cdot m_1 \cdot \left[ \frac{(m_2 \cdot a_2 + m_2 \cdot g)}{m_3} - g \right]$$

$$\Leftrightarrow \ (2 \cdot m_1 - m_2) \cdot g - \frac{4 \cdot m_1}{m_3} \cdot (m_2 - m_3) \cdot g = \left[ (4 \cdot m_1 + m_2) + \frac{4 \cdot m_1 \cdot m_2}{m_3} \right] \cdot a_2$$

$$\Leftrightarrow \; a_2 = \frac{\left[(2 \cdot m_1 - m_2) \cdot g - \frac{4 \cdot m_1}{m_3} \cdot (m_2 - m_3) \cdot g\right]}{(4 \cdot m_1 + m_2) + \frac{4 \cdot m_1 \cdot m_2}{m_3}}$$

$$= \frac{\left[(2 \cdot 5.65 - 2.30) \cdot 9.81 - \frac{4 \cdot 5.65}{4.25} \cdot (2.30 - 4.25) \cdot 9.81\right]}{(4 \cdot 5.65 + 2.30) + \frac{4 \cdot 5.65 \cdot 2.30}{4.25}}$$

$$= 5.12 \text{ m/s}^2$$

The acceleration $a_3$ for the pile of bricks is found as follows:

$$(m_2 \cdot a_2 + m_2 \cdot g) - m_3 \cdot g = m_3 \cdot a_3$$

$$\Leftrightarrow \; a_3 = \frac{(m_2 \cdot a_2 + m_2 \cdot g)}{m_3} - g$$

$$= \frac{(2.30 \cdot 5.12 + 2.30 \cdot 9.81)}{4.25} - 9.81$$

$$= -1.73 \text{ m/s}^2$$

Finally, the acceleration $a_1$ for the bucket filled with black clay is calculated by using the fourth constraint:

$$a_1 = -2 \cdot (a_2 + a_3)$$

$$= -2 \cdot (5.12 - 1.73)$$

$$= -6.77 \text{ m/s}^2$$

For the given masses, this means that the bucket of clay accelerates downwards rather quickly, whereas the iron ball shoots upwards. Similar to the bucket, the stack of bricks is being lowered, but at a much lower rate than the bucket. If Tony wishes to establish a more stable system, he could, for instance, attach a heavier iron ball to the second pulley. For example, a combined pulley system with an iron ball having the same mass as the bucket of clay ($m_2 = m_1 = 5.65$ kg) accelerates as follows: $a_1 = -5.06$ m/s$^2$ (bucket of clay), $a_2 = -0.302$ m/s$^2$ (iron ball), and $a_3 = 2.83$ m/s$^2$ (pile of bricks).

# Exercise 8

## ***Problem Statement***

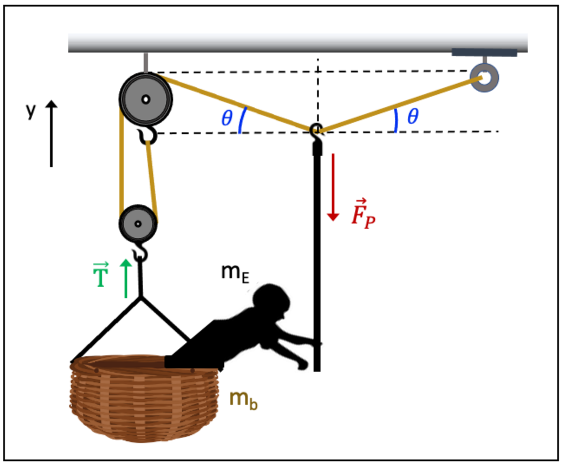


***Figure 11***

Harper recently attached a pulley system to the ceiling of her garage in Utah, the United States, and her 7-year-old son Ethan ($m_E = 24.5$ kg) asks his mom whether he can sit in the basket ($m_b = 6.50$ kg) that hangs from the pulley. As Harper knows that the system can easily withstand a weight of 500 N, she reckons it is safe to put her son in the basket. Mischievous as he is, Ethan grabs a stick with a hook on one end and pulls down on the rope that connects the pulley with the metal ring attached to the ceiling. (1) If the rope makes a $\theta = 25.0°$ angle with the horizontal on both sides from the point where the stick touches the rope, with what force $\vec{F}_P$ is Ethan pulling the rope? (2) Given a 2.50 m distance between the pulley and the metal ring, by how much did Ethan manage to pull himself up? (3) Suppose that Ethan increases his pulling force by 35%, which angle does the rope now make with the horizontal? (4) By what distance is Ethan now moving upwards towards the ceiling? Assume for each part of the question that Ethan is hanging still and holds his respective position.

## ***Solution***

(1) Let us first determine the tension in the various segments of the rope. Since the magnitude of the tension in the cable between the basket and the pulley is equal to T, the two rope segments between the two pulleys each experience a tension of $\frac{T}{2}$. This means that the tension in the rope between the pulley and the metal ring also measures $\frac{T}{2}$.

To find the force $\vec{F}_P$, we need to know the value of T, which we can calculate by identifying the forces acting on the system "basket plus Ethan". As per Newton's third law, the hook to which the basket is attached is exerting an upward force on "basket plus Ethan" due to their gravitational weight. In addition, the rope which Ethan is pulling downwards is also exhibiting an upward force on "basket plus Ethan" as a reaction to the downwards force $\vec{F}_P$ exerted on the rope. In fact, as there are two rope segments—one to the left of the point where the hook touches the rope and one to the right of that point—this upward force actually consists of two forces. With the knowledge that the net force is equal to zero, we can write Newton's second law as follows:

$$\vec{F}_{net} = \vec{T} + \frac{\vec{T}}{2} + \frac{\vec{T}}{2} + \vec{F}_G = \vec{0}$$

$$\Leftrightarrow \; T + \frac{T}{2} \cdot \sin\theta + \frac{T}{2} \cdot \sin\theta - (m_b + m_E) \cdot g = 0$$

$$\Leftrightarrow\ T \cdot [1 + \sin\theta] = (m_b + m_E) \cdot g$$

$$\Leftrightarrow\ T = \frac{(m_b + m_E) \cdot g}{1 + \sin\theta}$$

$$= \frac{(6.50 + 24.5) \cdot 9.81}{1 + \sin(25.0°)}$$

$$= 214 \text{ N}$$

Because Ethan is holding his position, there is no vertical movement, which implies that at the point where the hook touches the rope the two forces pointing upwards equal the force with which Ethan is pulling the rope:

$$\vec{F}_{net} = \frac{\vec{T}}{2} + \frac{\vec{T}}{2} + \vec{F}_P = \vec{0}$$

$$\Leftrightarrow\ \frac{T}{2} \cdot \sin\theta + \frac{T}{2} \cdot \sin\theta - F_P = 0$$

$$\Leftrightarrow\ F_P = T \cdot \sin\theta$$

$$= 214 \cdot \sin(25.0°)$$

$$= 90.3 \text{ N}$$

(2) When the rope makes an angle of 25.0° with the horizontal at both sides of the point of contact, the total length L of the rope between the pulley and the metal ring becomes (whereby the horizontal distance between the metal ring and the pulley is equal to $d = 2.50$ m):

$$L = 2 \cdot \left[\frac{\frac{d}{2}}{\cos\theta}\right] = 2 \cdot \left[\frac{1.25}{\cos(25.0°)}\right] = 2.76 \text{ m}$$

In other words, the rope has gotten $s = L - d = 2.76 - 2.50 = 0.258$ m or 25.8 cm longer. Since the rope in the *vertical* direction consists of two segments running parallel to each other, the distance with which Ethan gains height is equal to the total displacement (not the distance!) in the vertical direction:

$$\Delta y = \frac{s}{2} = \frac{25.8}{2} = 12.9 \text{ cm}$$

(3) If Ethan increase his pulling strength by 35%, the magnitude of the force becomes $F_P' = F_P \cdot 1.35 = 90.3 \cdot 1.35 = 122$ N. From part (1), we know that:

$$\begin{cases} F_P = T \cdot \sin\theta \\ \\ T + F_P = (m_b + m_E) \cdot g \end{cases}$$

When replacing T in the second equation by the expression for T obtained from the first equation, we find an expression for the angle $\theta$:

$$\frac{F_P}{\sin\theta} + F_P = (m_b + m_E) \cdot g$$

$$\Leftrightarrow \ \sin\theta = \frac{F_P}{(m_b + m_E) \cdot g - F_P}$$

$$\Leftrightarrow \ \theta = \sin^{-1}\left[\frac{F_P}{(m_b + m_E) \cdot g - F_P}\right]$$

Inserting the new value $F_P' = 122$ N, we find the new angle $\theta'$ that the rope makes with the horizontal:

$$\theta' = \sin^{-1}\left[\frac{F_P'}{(m_b + m_E) \cdot g - F_P'}\right] = \sin^{-1}\left[\frac{122}{(6.50 + 24.5) \cdot 9.81 - 122}\right] = 42.0°$$

(4) The length of the rope between the metal ring and the pulley now increases from $d = 2.50$ m to:

$$L' = 2 \cdot \left[\frac{\frac{d}{2}}{\cos\theta'}\right] = \frac{2.50}{\cos(42.0°)} = 3.37 \text{ m}$$

This time, the rope segment between the metal ring and the pulley grows longer by a distance of $s' = L' - d = 3.37 - 2.50 = 0.866$ m or 86.6 cm. The vertical displacement of Ethan towards the ceiling then becomes:

$$\Delta y' = \frac{s'}{2} = \frac{86.6}{2} = 43.3 \text{ cm}$$

# Exercise 9

## *Problem Statement*

Robert ($m_R$ = 66.0 kg) is asked to play a short intermezzo of $t$ = 35.0 s on his professional grand piano ($m_p$ = 317 kg) during a festival of classical music in the Brucknerhaus Linz concert hall in Linz, Austria, under some unusual circumstances. Robert will start playing on top of an 18.0 m-long incline, which makes a 23.5° angle with the horizontal, and while he is gradually speeding up the tempo of his musical intermezzo, he is simultaneously being lowered sideways with an ever increasing velocity towards the bottom of the incline. Behind the stage there is an integrated pulley system that has to coordinate Robert's act (see Fig. 12). If the mass of the counterweight B is equal to $m_B$ = 250 kg, what should be the mass of counterweight A, so that Robert arrives at the bottom of the incline precisely 35.0 s after he started playing his first note?

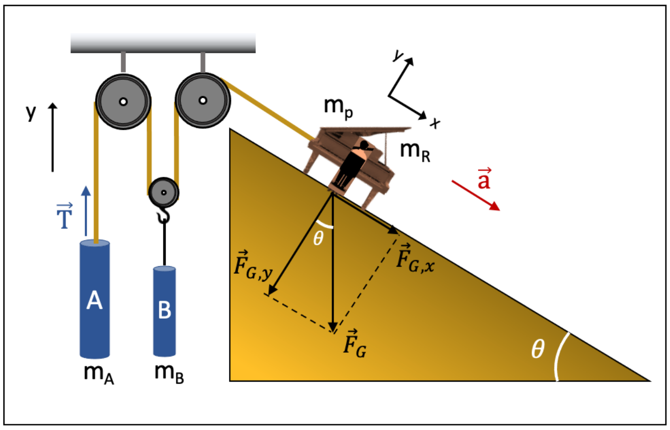


***Figure 12***

## *Solution*

From the constraint that Robert must cover the distance of 18.0 m in a time window of 35.0 s at an increasing velocity we can calculate the acceleration of the system "piano plus Robert":

$$x(t) = x_0 + v_{0x} \cdot t + \frac{a}{2} \cdot t^2$$

$$\Leftrightarrow \; 18.0 = 0 + 0 \cdot t + \frac{a}{2} \cdot 35.0^2$$

$$\Leftrightarrow \; a = \frac{2 \cdot 18.0}{35.0^2}$$

$$= 0.0294 \text{ m/s}^2$$

Using Newton's second law, we can write the appropriate equations for the counterweight A, the counterweight B, and the system "piano plus Robert" (in the x-direction), respectively:

$$\begin{cases} T - m_A \cdot g = m_A \cdot a_1 \\ 2 \cdot T - m_B \cdot g = m_B \cdot a_2 \\ -T + (m_p + m_R) \cdot g \cdot \sin\theta = (m_p + m_R) \cdot a \end{cases}$$

From the last equation we can calculate the magnitude of the tension $\vec{T}$:

$$T = (m_p + m_R) \cdot (g \cdot \sin\theta - a) = (317 + 66.0) \cdot [9.81 \cdot \sin(23.5°) - 0.0294] = 1,487 \text{ N}$$

Based on the second equation, we find a value for the acceleration of the counterweight B:

$$a_2 = \frac{2 \cdot T}{m_B} - g = \left(\frac{2 \cdot 1,487}{250}\right) - 9.81 = 2.09 \text{ m/s}^2$$

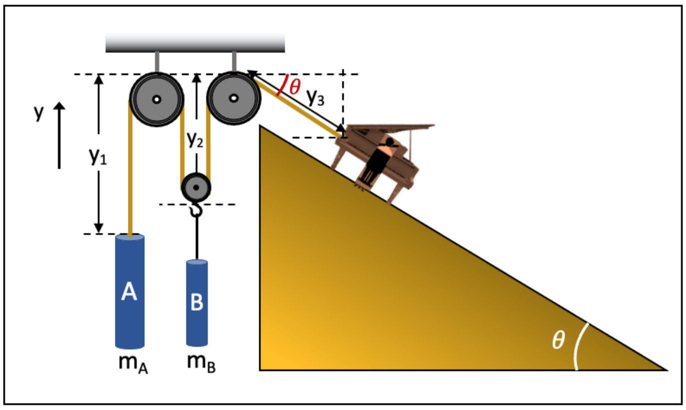


***Figure 13***

At this point, we need another constraint to calculate the acceleration of the counterweight A. Since the length of the rope remains constant, we can build a constraint by considering the vertical component of our coordinate system, i.e., the y-direction. Fig. 13 then shows that the following sum is always equal to some constant k:

$$y_1 + 2 \cdot y_2 + y_3 \cdot \sin\theta = k$$

Taking the second derivative of the above equation with respect to time gives us a constraint in terms of the various accelerations, whereby $a_3 = -a$ because we want the piano to move downwards:

$$a_1 + 2 \cdot a_2 - a \cdot \sin\theta = 0$$

The acceleration for the counterweight A then becomes:

$$a_1 = -2 \cdot a_2 + a \cdot \sin\theta = -2 \cdot 2.09 + 0.0294 \cdot \sin(23.5°) = -4.16 \text{ m/s}^2$$

Finally, based on the equation of Newton's second law, the mass of the counterweight A can be determined as follows:

$$m_A = \frac{T}{g + a_1} = \frac{1,487}{9.81 - 4.16} = 263 \text{ kg}$$

# Exercise 10

## ***Problem Statement***

You recently bought a mansion in the outskirts of Brno in the Czech Republic, and, as an architect, you're planning to design a new water fountain to put in the middle of the round square at the end of the driveway that leads to your house. Initially, the water in the fountain gradually flows down via a couple of steps, before falling down vertically. Since your daughter Madlenka just received a large Lego set for her eighth birthday from your brother Petr, you're building a miniature fountain out of Lego bricks, using weights, ropes, and pulleys to model the flow of water. If you would like the water to fall down from the last step at a pace of $a = 6.2$ m/s$^2$ (represented by the acceleration of weight 5 in Fig. 14), under what angle $\theta$ should you build the incline at the end of the last step? Assume the weights have the following masses: $m_1 = 1.5$ kg, $m_2 = 2.5$ kg, $m_3 = 3.5$ kg, $m_4 = 4.5$ kg, $m_5 = 5.5$ kg, and $m_6 = 6.5$ kg.

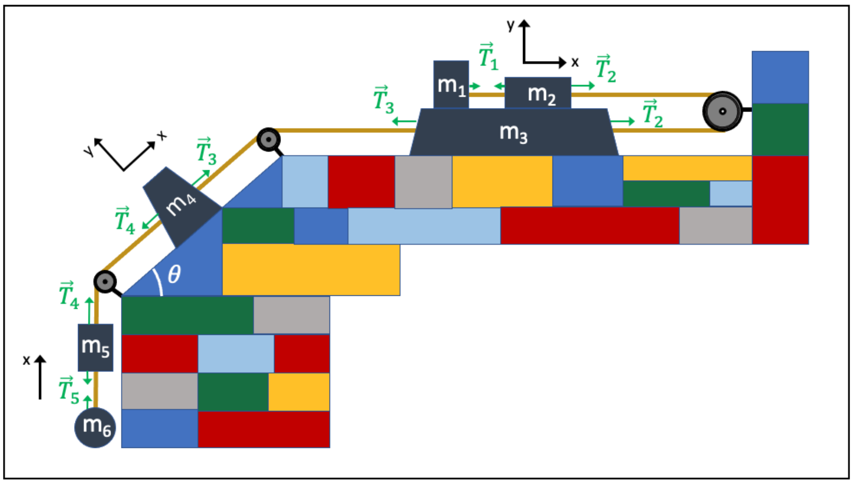


***Figure 14***

## ***Solution***

We can write the equation of Newton's second law for each of the six masses in the following way:

$$
\begin{cases}
m_1 \cdot a_1 = T_1 & m_4 \cdot a_4 = T_3 - T_4 - m_4 \cdot g \sin\theta \\
m_2 \cdot a_2 = -T_1 + T_2 & m_5 \cdot a_5 = T_4 - T_5 - m_5 \cdot g \\
m_3 \cdot a_3 = -T_3 + T_2 & m_6 \cdot a_6 = T_5 - m_6 \cdot g
\end{cases}
$$

As all of the weights are connected by ropes, the magnitude of the acceleration of all the weights is identical. However, the direction is not. As you would like the water to flow from the right to the left down the steps, the acceleration of weight 3, 4, 5, and 6 is negative. Under this scenario, the acceleration of the weights 1 and 2 must be positive. We can write these constraints as follows:

$$
\begin{cases}
a_1 = a_2 \\
a_3 = -a_1 \\
a_3 = a_4 = a_5 = a_6
\end{cases}
$$

If we write the equations of Newton's second law in terms of the acceleration of weight 1 ($a_1$), we obtain the following:

$$\begin{cases} m_1 \cdot a_1 = T_1 & m_4 \cdot a_1 = -T_3 + T_4 + m_4 \cdot g \sin\theta \\ m_2 \cdot a_1 = -T_1 + T_2 & m_5 \cdot a_1 = -T_4 + T_5 + m_5 \cdot g \\ m_3 \cdot a_1 = T_3 - T_2 & m_6 \cdot a_1 = -T_5 + m_6 \cdot g \end{cases}$$

By replacing $T_1$ in the equation of weight 2 by the expression for $T_1$ obtained from the equation of weight 1, we get an expression for $T_2$, which we can in turn use to replace $T_2$ in the equation of weight 3, and so on. If we apply this substitution exercise for all the weights, we eventually get the following equation:

$$m_6 \cdot a_1 = -m_1 \cdot a_1 - m_2 \cdot a_1 - m_3 \cdot a_1 - m_4 \cdot a_1 + m_4 \cdot g \sin\theta - m_5 \cdot a_1 + m_5 \cdot g + m_6 \cdot g$$

We can now write an expression for the angle $\theta$. To calculate the value of $\theta$, we use $a_1 = a = 6.2$ m/s$^2$, since the magnitude of all the accelerations is the same:

$$\begin{aligned} \sin\theta &= \frac{[(m_1 + m_2 + m_3 + m_4 + m_5 + m_6) \cdot a_1 - (m_5 + m_6) \cdot g]}{m_4 \cdot g} \\ \Leftrightarrow \quad \theta &= \sin^{-1}\left( \frac{[(m_1 + m_2 + m_3 + m_4 + m_5 + m_6) \cdot a_1 - (m_5 + m_6) \cdot g]}{m_4 \cdot g} \right) \\ &= \sin^{-1}\left( \frac{[(1.5 + 2.5 + 3.5 + 4.5 + 5.5 + 6.5) \cdot 6.2 - (5.5 + 6.5) \cdot 9.81]}{4.5 \cdot 9.81} \right) \\ &= 45^\circ \end{aligned}$$

# Exercise 11

## *Problem Statement*

Ashvin ($m_1 = 62.5$ kg) and Opaline ($m_2 = 55.8$ kg) are trying our their brand new wingsuits above the beaches of Port Louis on the Island of Mauritius in preparation of an upcoming skydiving event. Both these daredevils board a separate plane, and Opaline jumps out of her airplane first at a higher altitude than Ashvin. She quickly attains a terminal (i.e., constant) velocity of $v_{0,2} = 27.0$ m/s and descends vertically. At the moment when Opaline reaches the altitude of Ashvin's airplane, which is flying horizontally at $v_{plane} = 65.2$ km/h, Ashvin launches himself from his plane with his arms held close to his body. After a couple of seconds of free fall, Ashvin bumps into Opaline with a velocity of $\vec{v}_{0,1} = 18.1 \cdot \vec{i}_x + 54.0 \cdot \vec{i}_y$ m/s. Right after the collision, Opaline finds herself at the position $\vec{r}_{f,2} = 104 \cdot \vec{i}_x + 170 \cdot \vec{i}_y$ m. If the acceleration vector $\vec{a}$ makes a $\theta = 33.8°$ angle with the vertical, what is the magnitude of the force of impact on Opaline ($\vec{F}_{21}$)?

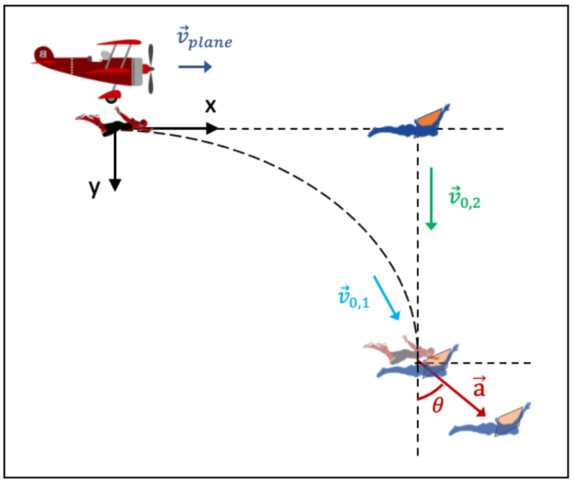


***Figure 15***

## *Solution*

What we first have to do is to find the position of the collision, for which we need to identify the time $t_{jump}$ it takes Ashvin to reach Opaline:

$$v_{0,1y} = v_0 + g \cdot t_{jump}$$

$$\Leftrightarrow \; t_{jump} = \frac{v_{0,1y} - v_0}{g} = \frac{54.0 - 0}{9.81} = 5.50 \text{ s}$$

The x- and y-component of the position vector $\vec{r}_0$ right before the moment when the skydivers hit each other are calculated as follows:

$$\begin{cases} x_0 = v_{0,1x} \cdot t_{jump} & y_0 = \frac{g}{2} \cdot t_{jump}^2 \\ \quad = 18.1 \cdot 5.50 & \quad = \frac{9.81}{2} \cdot 5.50^2 \\ \quad = 99.6 \text{ m} & \quad = 149 \text{ m} \end{cases}$$

Bearing in mind that the acceleration vector $\vec{a}$ makes a 33.8° angle with the vertical, we can now construct the following two equations of motion for Opaline during the time $t_{col}$ of the collision:

$$\begin{cases} x_{f,2} = x_0 + v_{0,2x} \cdot t_{col} + \dfrac{a_x}{2} \cdot t_{col}^2 \\ \Leftrightarrow\ 104 = 99.6 + 0 \cdot t_{col} + \dfrac{a \cdot \sin(33.8°)}{2} \cdot t_{col}^2 \\ \\ y_{f,2} = y_0 + v_{0,2y} \cdot t_{col} + \dfrac{a_y}{2} \cdot t_{col}^2 \\ \Leftrightarrow\ 170 = 149 + 27.0 \cdot t_{col} + \dfrac{a \cdot \cos(33.8°)}{2} \cdot t_{col}^2 \end{cases}$$

Replacing $t_{col}^2$ in the second equation (y-direction) by the expression for $t_{col}^2$ obtained from the first equation (x-direction), we find the value of $t_{col}$ (remember to use the not rounded intermediate results during calculations):

$$170 = 149 + 27.0 \cdot t_{col} + \frac{a \cdot \cos(33.8°)}{2} \cdot \left[\frac{2 \cdot (104 - 99.6)}{a \cdot \sin(33.8°)}\right]$$

$$\Leftrightarrow\ t_{col} = \frac{21.4 - 4.31 \cdot \text{cotan}(33.8°)}{27.0} = 0.553 \text{ s}$$

By considering, for instance, the equation of motion in the x-direction, the magnitude of the acceleration $\vec{a}$ with which Opaline is being thrusted forward in the direction of 33.8° with the vertical becomes:

$$a = \frac{2 \cdot (104 - 99.6)}{t_{col}^2 \cdot \sin(33.8°)} = \frac{2 \cdot 4.31}{0.553^2 \cdot \sin(33.8°)} = 50.5 \text{ m/s}^2$$

The magnitude of the force $\vec{F}_{21}$ that acts on Opaline is found as follows:

$$F_{21} = m_2 \cdot a = 55.8 \cdot 50.5 = 2.82 \times 10^3 \text{ N}$$

This force of impact is equivalent to Opaline being hit by a mass of $m_{impact} = \frac{F_{21}}{g} = \frac{2.82 \times 10^3}{9.81} = 287$ kg, which is not a very safe situation. Due to Newton's third law, Opaline will also exert a force $F_{12}$ on Ashvin with equal magnitude but opposite in direction. In other words, the acceleration vector of Ashvin will point in the negative x- and y-direction with a magnitude of $a = \frac{F_{12}}{m_1} = \frac{2.82 \times 10^3}{62.5} = 45.1$ m/s$^2$.

# Exercise 12

## ***Problem Statement***

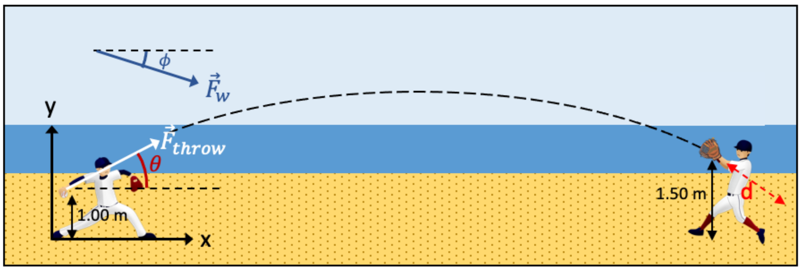


***Figure 16***

On a casual Wednesday afternoon, Nirmala and Harun are pitching some baseballs on Kemala Beach in Balikpapan, Indonesia. Nirmala is extending her right arm backwards, so that it is positioned 1.00 m above the ground, and launches the baseball ($m_b = 0.15$ kg) with an average force of $F_{throw} = 7.20$ N under an angle of $\theta = 55.0°$ with the horizontal over a distance of 1.50 m in $t_{throw} = 0.25$ s. At the same time when Nirmala is about to release the baseball, a wind of 14.2 kts kicks in at an angle of $\phi = 22.5°$ below the horizontal and generates a corresponding constant force of $F_w = 1.15$ N on the ball. How far backwards should Harun move his left hand—this is the distance $d$ in the same direction as the incoming baseball—when catching the ball 1.50 m above the ground, so that the equivalent mass upon impact is equal to $m_{impact} = 2.35$ kg?

## ***Solution***

In a first instance, let us determine the initial velocity with which Nirmala is flinging the baseball across the beach. Given that the magnitude of the acceleration $\vec{a}$ during the throw is equal to $a_{throw} = \frac{F_{throw}}{m_b} = \frac{7.20}{0.15} = 48.0 \text{ m/s}^2$, the baseball's initial speed is calculated to be:

$$v_{0,throw} = v_0 + a_{throw} \cdot t_{throw} = 0 + 48.0 \cdot 0.25 = 12.0 \text{ m/s}$$

Now, we want to find the airtime $t_{air}$ of the baseball right before Harun catches it, by solving the below equation of motion in the y-direction of the ball. Keep in mind that the magnitude of the acceleration $\vec{a}_w$ of the wind is equal to $a_{wind} = \frac{F_w}{m_b} = \frac{1.15}{0.15} = 7.67 \text{ m/s}^2$ and that the initial height $y_0$ is equal to $y_0 = y_{arm} + y_{throw} = 1.00 + 1.50 \cdot \sin(55.0°) = 2.23$ m . We can then write:

$$y(t_{air}) = y_0 + v_{0,y} \cdot t_{air} + \frac{a_y}{2} \cdot t_{air}^2$$

$$\Leftrightarrow \; y(t_{air}) = y_0 + (v_{0,throw} \cdot \sin\theta) \cdot t_{air} - \left[\frac{g + a_{wind} \cdot \sin\phi}{2}\right] \cdot t_{air}^2$$

$$\Leftrightarrow \; 1.50 = 2.23 + [12.0 \cdot \sin(55.0°)] \cdot t_{air} - \left[\frac{9.81 + 7.67 \cdot \sin(22.5°)}{2}\right] \cdot t_{air}^2$$

The physically meaningful ($t \geq 0$) solution for the above quadratic equation is $t_{air} = 1.61$ s. The x- and y-components of the velocity at which the baseball arrives in Harun's glove are equal to:

$$\begin{cases} v_x &= (v_{0,throw} \cdot \cos\theta) + (a_{wind} \cdot \cos\phi) \cdot t_{air} \\ &= [12.0 \cdot \cos(55.0°)] + [7.67 \cdot \cos(22.5°)] \cdot 1.61 \\ &= 18.3 \text{ m/s} \\ \\ v_y &= (v_{0,throw} \cdot \sin\theta) - (g + a_{wind} \cdot \sin\phi) \cdot t_{air} \\ &= [12.0 \cdot \sin(55.0°)] - [9.81 + 7.67 \cdot \sin(22.5°)] \cdot 1.61 \\ &= -10.7 \text{ m/s} \end{cases}$$

This gives a resultant velocity vector $\vec{v}_{catch}$ with a magnitude of:

$$v_{catch} = \sqrt{v_x^2 + v_y^2} = \sqrt{18.3^2 + (-10.7)^2} = 21.2 \text{ m/s}$$

Finally, we can calculate the distance $d$ by which Harun should extend his arm backwards upon catching the baseball. Bearing in mind that an equivalent mass of impact of $m_{impact} = 2.35$ kg corresponds to a stopping force of $F_{stop} = m_{impact} \cdot g = 2.35 \cdot 9.81 = 23.1$ N and that this force in turn corresponds to a deceleration $\vec{a}_{stop}$ of the baseball, whose magnitude is equal to $a_{stop} = \frac{F_{stop}}{m_b} = \frac{23.1}{0.15} = 154$ m/s$^2$, we find the distance $d$ as follows:

$$v_{stop}^2 - v_{catch}^2 = 2 \cdot a_{stop} \cdot d$$

$$\Leftrightarrow \; d = \frac{v_{stop}^2 - v_{catch}^2}{2 \cdot a_{stop}} = \frac{0^2 - 21.2^2}{2 \cdot (-154)} = 1.47 \text{ m}$$

# Exercise 13

## *Problem Statement*

Caleb is navigating his Sar 880V Cruiser (with a displacement mass of $m_{dis} = 5.50 \times 10^3$ kg) along the coast of Mayaro Bay, Trinidad and Tobago, with a speed of $v_0 = 13.35$ kts (1 knot =1.852 km/h) heading north towards Ortoire where he will attend a Sunday brunch at his mother's house. One meter to the left of Caleb, two conches that he collected during previous travels are suspended from the same rope, which, in turn, is attached to an aluminum bar. The conch hanging higher is called Lobatus gigas or queen conch ($m_{LG} = 2.50$ kg) and the one below is the Charonia tritonis or giant triton ($m_{CT} = 3.80$ kg). While Caleb is gazing at the Atlantic Ocean through his binoculars, he suddenly spots a blue whale in the northeastern direction at a distance of about 250 m. As he plans to change course and accelerate ($a_i = 0.8226$ m/s$^2$) for 17.8 s until he is 85.0 m away from the whale, Caleb has to take into account an ocean current (southbound) which causes his boat to experience a constant force of $F_{cur} = 1,925$ N. Right at the moment when he sets off in the appropriate direction, the two conches no longer hang vertically but each make a certain angle with the vertical, as shown in Fig. 17. What is the value of these two angles $\alpha$ and $\beta$?

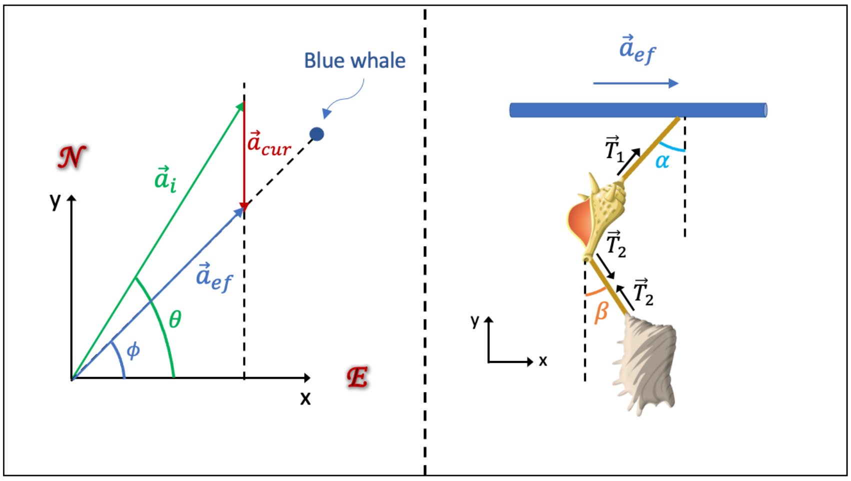


*Figure 17*

## *Solution*

The magnitude of the initial velocity $v_0$ expressed in m/s is equal to $v_0 = \frac{13.35*1.852}{3.6} = 6.87$ m/s and that of the acceleration caused by the southern current is $a_{cur} = \frac{F_{cur}}{m_{dis}} = \frac{1,925}{5,500} = 0.350$ m/s$^2$. Also remember that the direction northeast corresponds to an angle of $\phi = 45.0°$ with the horizontal. Keep furthermore in mind that the acceleration $a_i = 0.8226$ m/s$^2$ is the acceleration with respect to still water, not the actual acceleration $\vec{a}_{ef}$ of the boat. In a first step, we write the equation of motion in the x-and y-direction of the boat in order to identify the direction $\theta$ in which Caleb needs to be headed if he wishes to end up at $d = 250 - 85.0 = 165$ m from the blue whale:

$$\begin{cases} \Delta x & = v_{0,x} \cdot t + \dfrac{a_x}{2} \cdot t^2 \\ \Leftrightarrow\ d \cdot \cos\phi & = x\,[v_0 \cdot \cos\theta] \cdot t + \left[\dfrac{a_i \cdot \cos\theta}{2}\right] \cdot t^2 \\ \Leftrightarrow\ 165 \cdot \cos(45.0°) & = [6.87 \cdot \cos\theta] \cdot 17.8 + \left[\dfrac{0.8226 \cdot \cos\theta}{2}\right] \cdot 17.8^2 \end{cases}$$

$$\begin{cases} \Delta y & = & v_{0,y} \cdot t + \dfrac{a_y}{2} \cdot t^2 \\ \\ \Leftrightarrow \; d \cdot \sin\phi & = & \left[\dfrac{v_0 \cdot \cos\theta}{\cos\phi}\right] \cdot \sin\phi \cdot t + \left[\dfrac{a_i \cdot \sin\theta - a_{cur}}{2}\right] \cdot t^2 \\ \\ \Leftrightarrow \; 165 \cdot \sin(45.0^\circ) & = & \left[\dfrac{6.87 \cdot \cos\theta}{\cos(45.0^\circ)}\right] \cdot \sin(45.0^\circ) \cdot 17.8 + \left[\dfrac{0.8226 \cdot \sin\theta - 0.350}{2}\right] \cdot 17.8^2 \end{cases}$$

Given that the above equations of motion are written in terms of the actual path ($\vec{a}_{ef}$) that Caleb's boat will take (from the perspective of someone standing ashore, i.e., an stationary reference frame), the initial velocity $\vec{v}_0$ must equally be rescaled accordingly (note hereby that although the velocity vectors are not drawn in Fig. 17, they run parallel to the acceleration vectors $\vec{a}_i$ and $\vec{a}_{ef}$, respectively), which explains the expression $\frac{v_0 \cdot \cos\theta}{\cos\phi}$ in the equation of the y-direction. If we take, for instance, the equation of the x-direction, we can calculate the angle $\theta$ as follows:

$$165 \cdot \cos(45.0^\circ) = (6.87 \cdot \cos\theta) \cdot 17.8 + \left[\frac{0.8226 \cdot \cos\theta}{2}\right] \cdot 17.8^2$$

$$\Leftrightarrow \quad \cos\theta = \frac{165 \cdot \cos(45.0^\circ)}{(6.87 \cdot 17.8) + \left(\frac{0.8226}{2} \cdot 17.8^2\right)}$$

$$\Leftrightarrow \quad \theta = 62.5^\circ$$

As an alternative, the angle $\theta$ could have also been found with the assistance of the law of sines ("$\frac{a_i}{\sin(135^\circ)} = \frac{a_{cur}}{\sin(\theta-\phi)} = \frac{a_{ef}}{\sin(90^\circ-\theta)}$"), or simply by applying vector addition to the different acceleration vectors, instead of writing the equations of motion.

In view of the fact that the x-component of both $\vec{a}_i$ and $\vec{a}_{ef}$ is the same, the magnitude of the effective acceleration $\vec{a}_{ef}$ then becomes $a_{ef} = \frac{a_i \cdot \cos\theta}{\cos\phi} = \frac{0.8226 \cdot \cos(62.5^\circ)}{\cos(45.0^\circ)} = 0.537$ m/s$^2$.

Now that we know the value of the effective acceleration of Caleb's boat, we can shift our focus to the suspended conches. Applying Newton's second law to the x-and y-direction for the queen conch and the giant triton, respectively, we get the following equations:

| Queen conch (upper) | | Giant triton (lower) | |
|---|---|---|---|
| $x:$ | $T_1 \cdot \sin\alpha + T_2 \cdot \sin\beta - m_{LG} \cdot a_{ef} = 0$ | $x:$ | $-T_2 \cdot \sin\beta - m_{CT} \cdot a_{ef} = 0$ |
| $y:$ | $T_1 \cdot \cos\alpha - T_2 \cdot \cos\beta - m_{LG} \cdot g = 0$ | $y:$ | $T_2 \cdot \cos\beta - m_{CT} \cdot g = 0$ |

Note that although the boat accelerates ($\vec{a}_{ef}$) to the right, the force experienced by the conches

($\vec{F} = m_{conch} \cdot \vec{a}_{ef}$) points to the left (not drawn in Fig. 17), which explains the minus sign in front of ($m_{LG} \cdot a_{ef}$) and ($m_{CT} \cdot a_{ef}$), respectively. The force $\vec{F}$, which is felt by an observer, i.e., the conches, within an accelerating (non-inertial) framework, i.e., the boat, is a so-called pseudo-force and is the result of the relative motion of this non-inertial framework with respect to an inertial framework, i.e., the ground. Newton's laws do *not* hold in non-inertial frameworks, but with the introduction of pseudo-forces they can nevertheless be applied.

Replacing $T_2$ in the giant triton's x-equation by the expression for $T_2$ obtained from its y-equation, we find the value for the angle $\beta$:

$$-\left[\frac{m_{CT} \cdot g}{\cos\beta}\right] \cdot \sin\beta - m_{CT} \cdot a_{ef} = 0$$

$$\Leftrightarrow \tan\beta = -\frac{a_{ef}}{g}$$

$$\Leftrightarrow \beta = \tan^{-1}\left(-\frac{a_{ef}}{g}\right) = \tan^{-1}\left(-\frac{0.537}{9.81}\right) = -3.13°$$

The value for $T_2$ then becomes $T_2 = \frac{m_{CT} \cdot g}{\cos\beta} = \frac{3.80 \cdot 9.81}{\cos(-3.13°)} = 37.3$ N. Plugging these values into the queen conch's y-direction and subsequently replacing $T_1$ in its x-direction by the expression for $T_1$ obtained from the y-direction, we can calculate the angle $\alpha$ by writing the queen conch's x-direction as follows:

$$\left[\frac{T_2 \cdot \cos\beta + m_{LG} \cdot g}{\cos\alpha}\right] \cdot \sin\alpha + T_2 \cdot \sin\beta - m_{LG} \cdot a_{ef} = 0$$

$$\Leftrightarrow \tan\alpha = \frac{m_{LG} \cdot a_{ef} - T_2 \cdot \sin\beta}{T_2 \cdot \cos\beta + m_{LG} \cdot g}$$

$$\Leftrightarrow \alpha = \tan^{-1}\left(\frac{m_{LG} \cdot a_{ef} - T_2 \cdot \sin\beta}{T_2 \cdot \cos\beta + m_{LG} \cdot g}\right)$$

$$= \tan^{-1}\left(\frac{2.50 \cdot 0.537 - 37.3 \cdot \sin(-3.13°)}{37.3 \cdot \cos(-3.13°) + 2.50 \cdot 9.81}\right)$$

$$= 3.13°$$

At the very first instance of acceleration towards the blue whale, the conches make the same angle with an opposite sign. Even though the giant triton (the lower conch) has a greater mass ($m_{CT} = 3.80$ kg $> m_{LG} = 2.50$ kg) and thus experiences a larger pseudo-force F, it initially lags a little bit behind in terms of horizontal motion due to inertia. In a next moment, the giant triton will move to the left, triggering a further dynamical interplay of forces between the two conches.

# Exercise 14

## *Problem Statement*

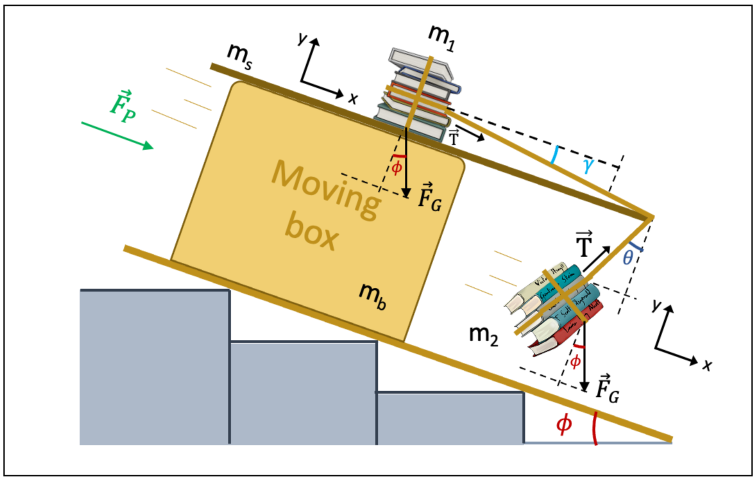


*Figure 18*

Zoe has been living in Tresses, France, for the past four years and is now moving to Bordeaux where she is starting a PhD in theoretical physics at the University of Bordeaux. Zoe is almost done packing and she just needs to put one final box ($m_b$) into her car. To avoid overburdening her back, Zoe has placed a ramp in front of her house, so she can slide the moving boxes towards her car. Lying on top of this last box, there is a shelf ($m_s = 2.8$ kg) on which two piles of books ($m_1 = 1.8$ kg and $m_2 = 0.60$ kg, respectively) are placed that she bound together with some rope. When Zoe places the box on the ramp, which makes an angle of $\phi = 18°$ with the ground, the second stack of books falls off the edge of the shelf and is dangling from the rope that is connected to the first pile of books. The rope between the first stack of books and the edge of the shelf now makes an angle of $\gamma = 6.5°$. Since the first pile is now starting to slide towards the edge, Zoe pushes the box down the ramp with a force $\vec{F}_P = 1.3 \times 10^2 \cdot \vec{i}_x$ N in order to keep the first pile on a fixed position on the shelf. What is the mass $m_b$ of the moving box? Assume that the shelf remains in place with respect to the moving box.

## *Solution*

In order for the first stack of books to remain steady on its position on the shelf, its acceleration in the x-direction must be equal to the net acceleration due to the force exerted by Zoe on the moving box plus any part that gravity plays. Let us in a first instance concentrate on the two piles of books. Based on Newton's second law, we write the following equations (we ignore the equation in the y-direction of the first stack, as it is irrelevant to our problem):

First pile

$$x: \quad T \cdot \cos\gamma + m_1 \cdot g \cdot \sin\phi = m_1 \cdot a_x$$

Second pile

$$x: \quad T \cdot \sin\theta + m_2 \cdot g \cdot \sin\phi = m_2 \cdot a_x$$

$$y: \quad T \cdot \cos\theta - m_2 \cdot g \cdot \cos\phi = 0$$

We now want to use the above equations to write an expression in terms of only one unknown variable, i.e., $a_x$, and therefore getting rid of the angle $\theta$ and the tension force $\vec{T}$. Relying on the trigonometric identity "$\cos^2\theta + \sin^2\theta = 1$", we can transform the second pile's y-equation as follows:

$$T \cdot (\sqrt{1 - \sin^2\theta}) - m_2 \cdot g \cdot \cos\phi = 0$$

$$\Leftrightarrow \quad \sin\theta = \sqrt{1 - \left(\frac{m_2 \cdot g \cdot \cos\phi}{T}\right)^2}$$

Plugging the above expression for $\sin\theta$ into the second pile's x-equation, we find an equation without the unknown variable $\theta$:

$$T \cdot \sqrt{1 - \left(\frac{m_2 \cdot g \cdot \cos\phi}{T}\right)^2} + m_2 \cdot g \cdot \sin\phi = m_2 \cdot a_x$$

$$\Leftrightarrow \; T^2 = m_2^2 \cdot a_x^2 - 2 \cdot m_2^2 \cdot g \cdot \sin\phi \cdot a_x + m_2^2 \cdot g^2$$

From the first pile's x-equation, we can write the following expression for $T^2$:

$$T^2 = \frac{m_1^2}{\cos^2\gamma} \cdot \left[a_x^2 - 2 \cdot g \cdot \sin\phi \cdot a_x + g^2 \cdot \sin^2\phi\right]$$

By combining the two previous equations we eliminate the variable T and, after some rearranging of the various terms, we end up with a quadratic equation that only depends on $a_x$:

$$\left[\frac{m_1^2}{\cos^2\gamma} - m_2^2\right] \cdot a_x^2 + \left[2 \cdot g \cdot \sin\phi \cdot \left(m_2^2 - \frac{m_1^2}{\cos^2\gamma}\right)\right] \cdot a_x + \left[g^2 \cdot \left(m_1^2 \cdot \frac{\sin^2\phi}{\cos^2\gamma} - m_2^2\right)\right] = 0$$

$$\Leftrightarrow \quad \left[\frac{1.8^2}{\cos^2(6.5°)} - 0.60^2\right] \cdot a_x^2 + \left[2 \cdot 9.81 \cdot \sin(18°) \cdot \left(0.60^2 - \frac{1.8^2}{\cos^2(6.5°)}\right)\right] \cdot a_x +$$

$$\left[9.81^2 \cdot \left(1.8^2 \cdot \frac{\sin^2(18°)}{\cos^2(6.5°)} - 0.60^2\right)\right] = 0$$

The physically relevant ($a_x > 0$ m/s$^2$ since the pile slides to the right) solution of the above quadratic equation is $a_x = 6.3$ m/s$^2$. Finally, applying Newton's second law to the system "moving box plus shelf plus the two piles of books" in the x-direction generates another equation, which in turn allows us to calculate the mass $m_b$ of the moving box:

$$F_P + (m_b + m_s + m_1 + m_2) \cdot g \cdot \sin\phi = (m_b + m_s + m_1 + m_2) \cdot a_x$$

$$\Leftrightarrow \; m_b = \frac{F_P}{a_x - g \cdot \sin\phi} - (m_s + m_1 + m_2) = \frac{1.3 \times 10^2}{6.3 - 9.81 \cdot \sin(18°)} - (2.8 + 1.8 + 0.60) = 34 \text{ kg}$$

# Exercise 15

## *Problem Statement*

Lixue ($m_L = 55.5$ kg) and Chaun ($m_C = 57.5$ kg) are practicing their trapeze act for the upcoming Lantern Festival in Tianshui (Gansu province), China. At one particular moment during their act, they both jump from opposite sides of the stage from a small platform 9.50 m above ground level onto the aluminum bar of their trapeze ($m_b = 2.10$ kg) and swing towards the middle. As a result, Lixue and Chaun provide their trapeze with an initial push of $\vec{F}_L = 172 \cdot \vec{i}_x$ N and $\vec{F}_C = -188 \cdot \vec{i}_x$ N, respectively. Both trapezes hang from the same height about $x_{trap} = 5.30$ m apart but the cables of Lixue's trapeze are 1.00 m longer ($s_L = 6.00$ m). If you know that both artists can extend their arms for an additional distance of $x_{arm} = 1.00$ m towards each other while swinging on their trapeze, do they manage to touch hands when reaching their farthest point in the horizontal direction? Assume that the origin of the coordinate system is located at the position of the aluminum bar of Lixue's trapeze when her trapeze is hanging vertically and still.

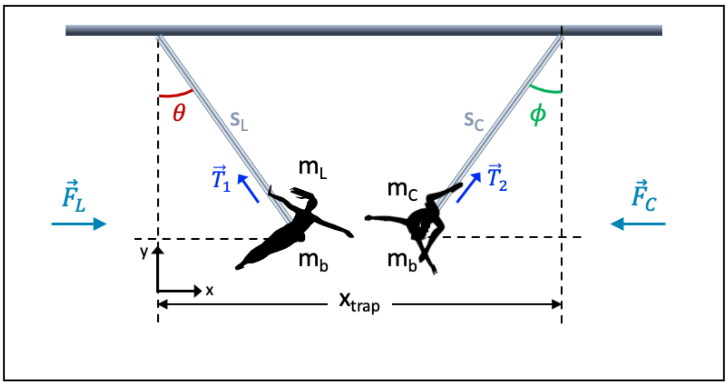


***Figure 19***

## *Solution*

First off, let us write down the equations of Newton's second law for each of the trapeze artists:

| Lixue (left) | Chaun (right) |
|---|---|
| $x: \quad -T_1 \cdot \sin\theta = F_{net,L}$ | $x: \quad T_2 \cdot \sin\phi = F_{net,C}$ |
| $y: \quad T_1 \cdot \cos\theta - (m_L + m_b) \cdot g = 0$ | $y: \quad T_2 \cdot \cos\phi - (m_C + m_b) \cdot g = 0$ |

Replacing the tension force in the x-equation by the expression for the tension obtained from the y-equation, we find an expression for the angle for both Lixue and Chaun:

| Lixue (left) | Chaun (right) |
|---|---|
| $x: \quad -\left[\frac{(m_L + m_b) \cdot g}{\cos\theta}\right] \cdot \sin\theta = F_{net,L}$ | $x: \quad \left[\frac{(m_C + m_b) \cdot g}{\cos\phi}\right] \cdot \sin\phi = F_{net,C}$ |
| $\Leftrightarrow \quad \theta = \tan^{-1}\left[-\frac{F_{net,L}}{(m_L + m_b) \cdot g}\right]$ | $\Leftrightarrow \quad \phi = \tan^{-1}\left[\frac{F_{net,C}}{(m_C + m_b) \cdot g}\right]$ |

As per Newton's third law, since Lixue (Chaun) initially exerted a force $\vec{F}_L$ ($\vec{F}_C$) on the trapeze, the trapeze (through the x-component of the tension force) is acting on Lixue (Chaun) with equal magnitude but opposite direction. In other words, $\vec{F}_{net,L} = -F_L \cdot \vec{i}_x$ ($\vec{F}_{net,C} = F_C \cdot \vec{i}_x$). Therefore, we obtain the following value for the angle $\theta$ ($\phi$):

Lixue (left)

$$\theta = \tan^{-1}\left[-\frac{-F_L}{(m_L + m_b)\cdot g}\right] = \tan^{-1}\left[-\frac{-172}{(55.5+2.10)\cdot 9.81}\right] = 16.9°$$

Chaun (right)

$$\phi = \tan^{-1}\left[\frac{F_C}{(m_C + m_b)\cdot g}\right] = \tan^{-1}\left[\frac{188}{(57.5+2.10)\cdot 9.81}\right] = 17.8°$$

We can now determine the position coordinates of both artists (bear in mind that the cables of Chaun's trapeze are 1.00 m shorter, so her initial position in the y-direction is equal to 1.00 m):

Lixue (left)

$$x_L = s_L \cdot \sin\theta = 6.00 \cdot \sin(16.9°) = 1.75 \text{ m}$$

$$y_L = s_L - s_L \cdot \cos\theta = 6.00 - 6.00 \cdot \cos(16.9°) = 0.260 \text{ m}$$

Chaun (right)

$$x_C = x_{trap} - (s_C \cdot \sin\phi) = 5.30 - [5.00 \cdot (17.8°)] = 3.77 \text{ m}$$

$$y_C = 1.00 + s_C - s_C \cdot \cos\phi = 1.00 + 5.00 - 5.00 \cdot \cos(17.8°) = 1.24 \text{ m}$$

Given that Lixue and Chaun are not on the same altitude, we use the Pythagorean theorem to calculate the distance $d$ between their hands:

$$d = \sqrt{(x_C - x_L)^2 + (y_C - y_L)^2} - 2 \cdot x_{arm} = \sqrt{(3.77-1.75)^2 + (1.24-0.260)^2} - 2 \cdot 1.00 = 0.247 \text{ m or } 24.7 \text{ cm}$$

It seems that the trapeze artists need more practice before featuring their act on the Lantern Festival.

# Exercise 16

## ***Problem Statement***

Ana Laura is a professor at the University of Montevideo, Uruguay, where she teaches the course quantum field theory, and in her spare time Ana Laura loves to build simplified models of planetary surface landers. Today, she is taking one of her latest models for a test flight and all seems to go well. As Ana Laura is guiding her lander ($m_{pl}$) vertically towards the ground, she simultaneously fires the four boosters a first time with a total force of $\vec{F_1} = 1.3 \times 10^3 \cdot \vec{i_y}$ N, providing the planetary lander with a net upwards acceleration $\vec{a}_1$, so that the lander slows down from $\vec{v}_{0,1} = -8.0 \cdot \vec{i_y}$ m/s to a velocity $\vec{v}$ over a time period $t_1 = 3.6$ s. Immediately afterwards, Ana Laura changes the power supplied by the boosters ($\vec{F_2}$) and after $t_2$ seconds, during which it has been displaced over a distance of $\Delta y_2 = 1.5$ m, the lander has obtained a final velocity of $\vec{v}_{f,2} = 4.4 \cdot \vec{i_y}$ m/s. If the ratio between the acceleration $a_1$ and $a_2$ is equal to 0.442 and given that, due to some technical constraints, the current model cannot accelerate faster than 5.0 m/s$^2$, (1) what is the mass $m_{pl}$ of Ana Laura's planetary lander, and (2) what is the magnitude of $\vec{F_2}$?

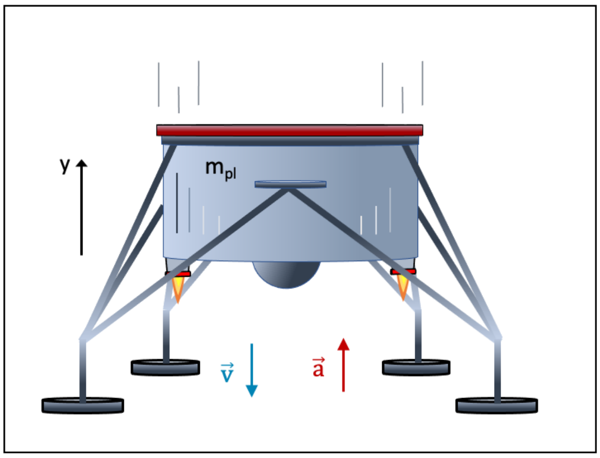


***Figure 20***

## ***Solution***

(1) With the provided data, we can write the following two equations of motion in the y-direction, one for each boost phase (note hereby that the final velocity $\vec{v}$ of boost phase 1 is the same as the initial velocity of boost phase 2):

| Boost Phase 1 | Boost Phase 2 |
| --- | --- |
| $v = v_{0,1} + a_1 \cdot t_1$ | $v_{f,2} = v + a_2 \cdot t_2$ |
| $\Leftrightarrow \quad v = -8.0 + a_1 \cdot 3.6$ | $\Leftrightarrow \quad 4.4 = v + a_2 \cdot t_2$ |

Replacing v in the equation of boost phase 1 by the expression for v obtained from the equation of boost phase 2, we find the following expression for $t_2$:

$$t_2 = \frac{12.4 - 3.6 \cdot a_1}{a_2}$$

Relying on another equation of motion for boost phase 2 and taking into account the above expression for $t_2$ as well as the fact that $a_1 = 0.442 \cdot a_2$, we find the following quadratic equation in terms of $a_1$:

$$\Delta y_2 = v_{0,2} \cdot t_2 + \frac{a_2}{2} \cdot t_2^2$$

$$\Leftrightarrow\ 1.5 = v \cdot \left[\frac{12.4 - 3.6 \cdot a_1}{a_2}\right] + \frac{a_2}{2} \cdot \left[\frac{12.4 - 3.6 \cdot a_1}{a_2}\right]^2$$

$$\Leftrightarrow\ 1.5 = (-8.0 + a_1 \cdot 3.6) \cdot \left[\frac{12.4 - 3.6 \cdot a_1}{a_2}\right] + \frac{a_2}{2} \cdot \left[\frac{12.4 - 3.6 \cdot a_1}{a_2}\right]^2$$

$$\Leftrightarrow\ 3.0 \cdot a_2 = 2 \cdot (-8.0 + a_1 \cdot 3.6) \cdot (12.4 - 3.6 \cdot a_1) + (12.4 - 3.6 \cdot a_1)^2$$

$$\Leftrightarrow\ \frac{3.0}{0.442} \cdot a_1 = 2 \cdot (-8.0 + a_1 \cdot 3.6) \cdot (12.4 - 3.6 \cdot a_1) + (12.4 - 3.6 \cdot a_1)^2$$

$$\Leftrightarrow\ (3.6^2 - 7.2 \cdot 3.6) \cdot a_1^2 + \left(24.8 \cdot 3.6 + 8 \cdot 7.2 - 7.2 \cdot 12.4 - \frac{3.0}{0.442}\right) \cdot a_1 - 8 \cdot 24.8 + 12.4^2 = 0$$

This equation has two solutions, i.e., $a_{(1,-)} = 2.6\ \text{m/s}^2$ and $a_{(1,+)} = 1.3\ \text{m/s}^2$, but since the acceleration $a_2 = \frac{a_{(1,-)}}{0.442} = \frac{2.6}{0.442} = 5.9\ \text{m/s}^2$ that corresponds with $a_{(1,-)}$ exceeds the technical constraint of $a \leq 5.0\ \text{m/s}^2$, we can exclude this solution. In the case of $a_{(1,+)}$, the acceleration $a_2 = \frac{a_{(1,+)}}{0.442} = \frac{1.3}{0.442} = 3.0\ \text{m/s}^2$ indeed respects this constraint.

Finally, applying Newton's second law to Ana Laura's planetary lander during boost phase 1 allows us to identify its mass $m_{pl}$:

$$F_1 - m_{pl} \cdot g = m_{pl} \cdot a_{(1,+)}$$

$$\Leftrightarrow\ m_{pl} = \frac{F_1}{g + a_{(1,+)}} = \frac{1.3 \times 10^3}{9.81 + 1.3} = 1.2 \times 10^2\ \text{kg}$$

(2) The magnitude of the force $\vec{F}_2$ of the second boost phase can be determined through Newton's second law:

$$F_2 - m_{pl} \cdot g = m_{pl} \cdot a_2$$

$$\Leftrightarrow\ F_2 = m_{pl} \cdot (g + a_2) = 1.2 \times 10^2 \cdot (9.81 + 3.0) = 1.5 \times 10^3\ \text{N}$$

# Exercise 17

## ***Problem Statement***

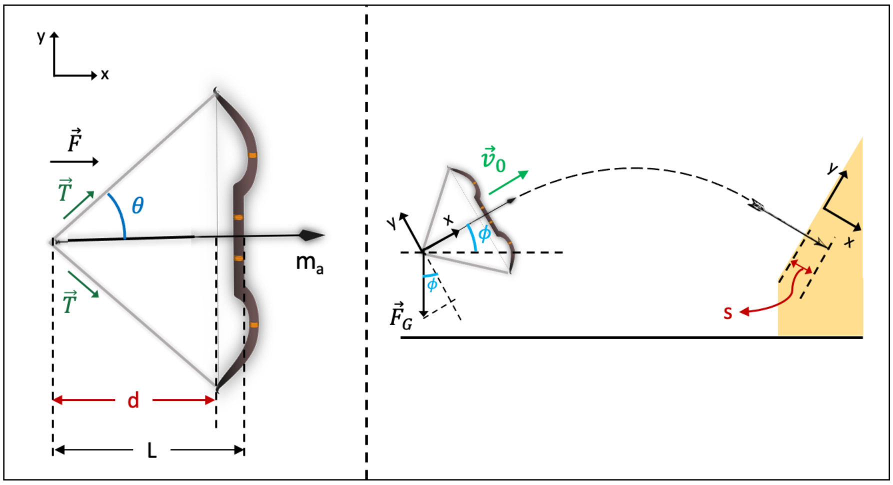


***Figure 21***

Amadou is writing his Bachelor's thesis at the University of Bamako, Mali, on the mechanics of the Quest Radical compound bow. In particular, Amadou is investigating whether a linear relationship exists between the segment $d$ of the draw length $L$ and the distance $s$ the arrow penetrates into a wooden block after being shot from a certain distance, whereby the mark is positioned at the same height as the bow. After some experimental testing, Amadou finds a relationship $s = \beta \cdot d$ with $\beta = 0.160$. He also know from previous research that a relationship exists between the segment $d$ and the magnitude of the tension force $\vec{T}$, i.e., $d = \gamma \cdot T$ with $\gamma = \frac{1}{450}$. If Amadou shoots an arrow ($m_a = 75.8$ g), which requires $t = 15.0$ ms to leave his bow with a velocity of $v_i = 79.1$ m/s, under an angle of $\phi = 22.0°$, (1) what is the magnitude of the tension force $\vec{T}$ in the string? (2) What angle $\theta$ does the string make with respect to the arrow? (3) What is the magnitude of the force $\vec{F}$ exerted by the string upon the arrow? (4) How deep does the arrow get stuck into the wooden block?

## ***Solution***

(1) At the moment when the arrow is being released and still subject to the force of the string, the following equation of motion provides us with the acceleration $a_i$ of the arrow:

$$v_i = v_0 + a_i \cdot t$$

$$\Leftrightarrow \; a_i = \frac{v_i - v_0}{t} = \frac{79.1 - 0}{0.015} = 5.27 \times 10^3 \text{ m/s}^2$$

The segment d of the bow is equal to the distance over which the arrow is being accelerated. This means that we can use the below equation of motion to calculate the length of d:

$$v_i^2 - v_0^2 = 2 \cdot a_i \cdot d$$

$$\Leftrightarrow \; d = \frac{v_i^2 - v_0^2}{2 \cdot a_i} = \frac{79.1^2 - 0^2}{2 \cdot 5.27 \times 10^3} = 59.3 \text{ cm}$$

The magnitude of the tension force $\vec{T}$ can now be calculated as follows:

$$d = \gamma \cdot T$$

$$\Leftrightarrow \; T = \frac{d}{\gamma} = 0.593 \cdot 450 = 267 \text{ N}$$

This tension force corresponds with a "draw weight" of $w = \frac{T}{g} = \frac{267}{9.81} = 27.2$ kg or 59.9 pounds.

(2) To determine the angle $\theta$ that the string makes with the arrow, we start from the equation of Newton's second law for the arrow (in the x-direction) when it is being shot under an angle of $\phi = 22.0°$:

$$2 \cdot T \cdot \cos\theta - m_a \cdot g \cdot \sin\phi = m_a \cdot a_i$$

$$\Leftrightarrow \;\; \cos\theta = \frac{m_a}{2 \cdot T} \cdot (a_i + g \cdot \sin\phi)$$

$$\Leftrightarrow \;\; \theta = \cos^{-1}\left[\frac{m_a}{2 \cdot T} \cdot (a_i + g \cdot \sin\phi)\right]$$

$$= \cos^{-1}\left[\frac{0.0758}{2 \cdot 267} \cdot (5.27 \times 10^3 + 9.81 \cdot \sin(22.0°))\right]$$

$$= 41.5°$$

(3) The magnitude of the force $\vec{F}$ that the string exerts upon the arrow is calculated as follows:

$$F = 2 \cdot T \cdot \cos\theta = 2 \cdot 267 \cdot \cos(41.5°) = 400 \text{ N}$$

(4) Based on Amadou's newly discovered linear relationship between the segment $d$ and the depth $s$, which represents the distance that the arrow penetrates the wooden block after being released, we find the following value for $s$:

$$s = \beta \cdot d = 0.160 \cdot 59.3 = 9.49 \text{ cm}$$

The next step that Amadou is going to take to fine-tune his linear relationship between the variables $d$ and $s$, consists of setting up new experiments—and hopefully identifying a new constant—to account for the hardness of the wood being used, which, according to his new hypothesis, inversely impacts the penetration depth $s$.

# Exercise 18

## ***Problem Statement***

As the first participants of the cross-country skiing event Skarverennet arrive in Ustaoset, Norway, Kjerstin ($m_K = 68.3$ kg) is enjoying the race from a higher altitude while paragliding above the scene. If we chose the origin of our coordinate system to coincide with Cafe Presttun with the y-axis pointing upwards and the x-axis eastwards, then Kjerstin finds herself at this moment at the position $\vec{r}_0 = 123 \cdot \vec{i}_x + 85.0 \cdot \vec{i}_y + 12.7 \cdot \vec{i}_z$ m with a velocity of $\vec{v}_0 = 5.33 \cdot \vec{i}_x - 2.20 \cdot \vec{i}_y + 0.860 \cdot \vec{i}_z$ m/s. For the next $t_w = 5.50$ s, Kjerstin experiences a wind gust that subjects her to an acceleration of $a = 2.33$ m/s$^2$ and points $\theta_1 = 31.1°$ upwards and $\theta_2 = 68.3°$ north of east. (1) If the gear that Kjerstin is wearing has a mass of $m_g = 5.80$ kg, what is the total force $\vec{F}$ that her seat is exerting upon her during the wind gust? (2) What distance did Kjerstin travel for the duration of the gust? (3) By how much is Kjerstin now farther away from or closer to Cafe Presttun with respect to her initial position?

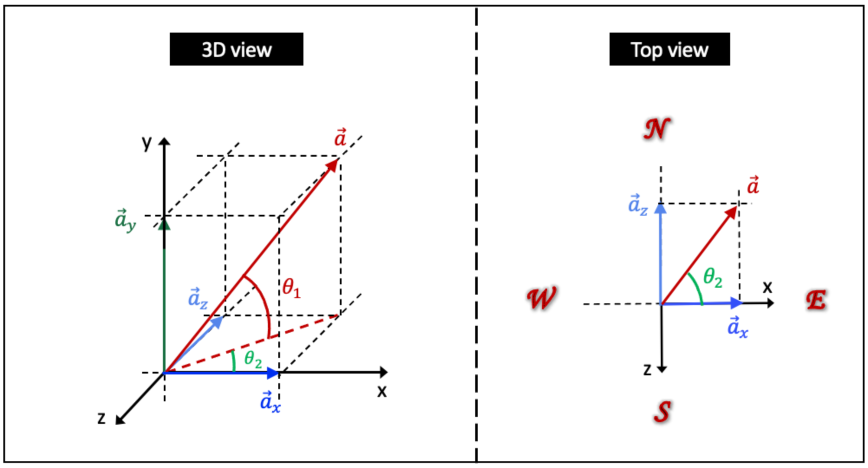


***Figure 22***

## ***Solution***

(1) We first write the magnitude of the three components of the acceleration vector $\vec{a}$:

| x-component | y-component | z-component |
|---|---|---|
| $a_x = a \cdot \cos\theta_1 \cdot \cos\theta_2$ | $a_y = a \cdot \sin\theta_1$ | $a_z = a \cdot \cos\theta_1 \cdot \sin\theta_2$ |
| $= 2.33 \cdot \cos(31.1°) \cdot \cos(68.3°)$ | $= 2.33 \cdot \sin(31.1°)$ | $= 2.33 \cdot \cos(31.1°) \cdot \sin(68.3°)$ |
| $= 0.737$ m/s$^2$ | $= 1.20$ m/s$^2$ | $= 1.85$ m/s$^2$ |

Note hereby that the orientation of $\vec{a}_z$ is towards the negative z-direction, i.e., $\vec{a}_z = -1.85 \cdot \vec{i}_z$ m/s$^2$. An acceleration of the paraglider in the positive x-direction means that Kjerstin experiences a pseudo-force (which exists due to a non-inertial framework, i.e., the paraglider, that is accelerating with respect to an inertial frame of reference, i.e., the ground) pushing her against her seat into the negative x-direction which, in turn, according to Newton's third law, pushes back on her in the opposite direction, i.e., the positive x-direction. Along the same reasoning, Kjerstin's seat pushes back on her with a component in the negative z-direction and the positive y-direction. The magnitude of the three components of the force vector $\vec{F}$ is calculated as follows:

| x-component | y-component | z-component |
|---|---|---|
| $F_x = (m_K + m_g) \cdot a_x$ | $F_y = (m_K + m_g) \cdot (a_y + g)$ | $F_z = (m_K + m_g) \cdot a_z$ |
| $= (68.3 + 5.80) \cdot 0.737$ | $= (68.3 + 5.80) \cdot (1.20 + 9.81)$ | $= (68.3 + 5.80) \cdot (1.85)$ |
| $= 54.6$ N | $= 816$ N | $= 137$ N |

Also the z-component $\vec{F}_z$ points in the negative z-direction. The magnitude of the total force $\vec{F}$ can be calculated as follows:

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{54.6^2 + 816^2 + 137^2} = 829 \text{ N}$$

The vector $\vec{F}$ points north of east under an angle of $\theta_{2F} = \tan^{-1}\left(\frac{F_z}{F_x}\right) = \tan^{-1}\left(\frac{137}{54.6}\right) = 68.3°$ and upwards with an angle equal to $\theta_{1F} = \tan^{-1}\left(\frac{F_y}{F_{xz}}\right) = \tan^{-1}\left(\frac{816}{148}\right) = 79.7°$, with $F_{xz} = \sqrt{F_x^2 + F_z^2} = \sqrt{54.6^2 + 137^2} = 148$ N.

(2) The total distance traveled by Kjerstin during the wind gust is in this case equal to the magnitude of the total displacement vector $\vec{s}$. Let us in a first instance determine the magnitude of the three components of $\vec{s}$:

| x-component | y-component | z-component |
|---|---|---|
| $\Delta x = v_{0x} \cdot t_w + \frac{a_x}{2} \cdot t_w^2$ | $\Delta y = -v_{0y} \cdot t_w + \frac{a_y}{2} \cdot t_w^2$ | $\Delta z = v_{0z} \cdot t_w - \frac{a_z}{2} \cdot t_w^2$ |
| $= 5.33 \cdot 5.50 + \left(\frac{0.737}{2}\right) \cdot 5.50^2$ | $= -2.20 \cdot 5.50 + \left(\frac{1.20}{2}\right) \cdot 5.50^2$ | $= 0.860 \cdot 5.50 - \left(\frac{1.85}{2}\right) \cdot 5.50^2$ |
| $= 40.5$ m | $= 6.10$ m | $= -23.3$ m |

Note that, with regard to the y-component, the acceleration $\vec{g}$ due to the gravitational force is not taken into account since it is already integrated into the paraglider's current motion. That is, its initial velocity in the y-direction ($\vec{v}_{0y} = -2.20 \cdot \vec{i}_y$ m/s) is already the result of the interplay between the gravitational downwards force and the upwards lift force due to air currents. The acceleration due to the gust of wind is an additional acceleration that alters Kjerstin's initial velocity and her course of motion. The magnitude of the displacement vector $\vec{s}$ is then found as follows:

$$s = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} = \sqrt{40.5^2 + 6.05^2 + (-23.3)^2} = 47.1 \text{ m}$$

(3) The initial distance that Kjerstin is away from Cafe Presttun is equal to the magnitude of the position vector $\vec{r}_0$, i.e., $r_0 = \sqrt{123^2 + 85.0^2 + 12.7^2} = 150$ m. The magnitude of Kjerstin's position vector $\vec{r}$ after the wind gust has passed is equal to:

$$r = \sqrt{(r_{0x} + \Delta x)^2 + (r_{0y} + \Delta y)^2 + (r_{0z} + \Delta z)^2}$$

$$= \sqrt{(123 + 40.5)^2 + (85.0 + 6.05)^2 + (12.7 - 23.3)^2}$$

$$= 187.4 \text{ m}$$

This means that Kjerstin is now $187.4 - 150 = 37.4$ m farther away from Cafe Presttun.

# Exercise 19

## *Problem Statement*

María Elena is a Venezuelan artist and she is invited to participate in an exhibition called "Formas y Figuras" ("Forms and Shapes") in the capital Caracas. For this occasion, María Elena selected one of her favourite works, i.e., an intricate piece of art that consists of various blocks made of the wood supplied by the Araguaney tree and carved in the shape of hexagons, nonagons, and dodecagons. The entire complex is held together by an interconnected web of ropes and miniature replications of statues made by other Venezuelan artists, which serve both to render homage to her fellow colleagues and to function as counterweights. María Elena is taking the elevator to her room in Hotel Tamanaco to pick up the last dodecagonal-shaped block and notices that she has gained 15% more weight relative to earlier that morning when she weighed herself in the bathroom—suppose hereby that a scale is installed in the elevator as an extra service for the guests. When riding the elevator back down, she releases for a moment the rope at the right-hand side of the block which causes the miniature statues to slide to the left with an acceleration of $a_{S*} = 0.450$ m/s$^2$ with respect to María Elena. Given a mass of $m_1 = 4.60$ kg, $m_2 = 3.30$ kg, $m_4 = 2.80$ kg, and $m_5 = 3.40$ kg for the other statues and the fact that the outer angle between two consecutive edges of a dodecagon is equal to $\theta = 30.0°$, what is the mass $m_3$ of statue number 3?

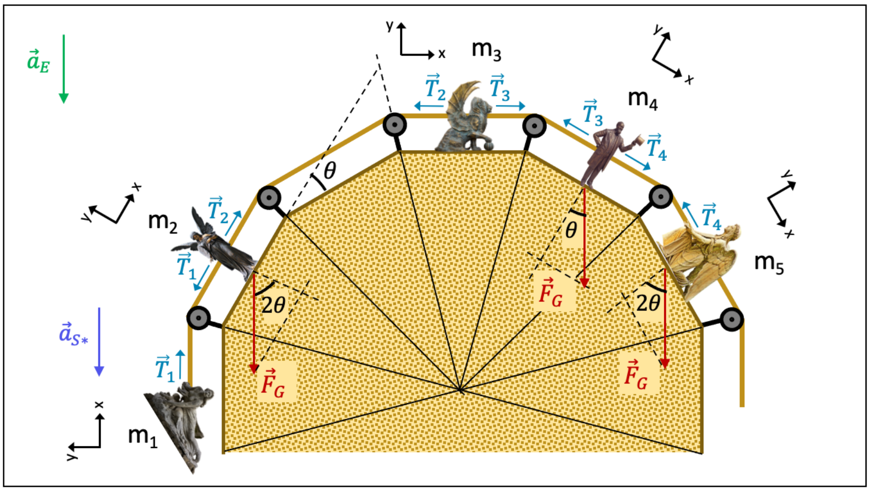


*Figure 23*

## *Solution*

In a first step, we wish to determine the magnitude of the elevator's acceleration $\vec{a}_E$. Based on Newton's second law, we can write the following equation, whereby $m_M$ and $F_N$ represent María Elena's mass and apparent weight, respectively:

$$\vec{F}_{net} = \vec{F}_N + \vec{F}_G$$

$$\Leftrightarrow \; m_M \cdot a_E = F_N - m_M \cdot g$$

$$\Leftrightarrow \; F_N = m_M \cdot a_E + m_M \cdot g$$

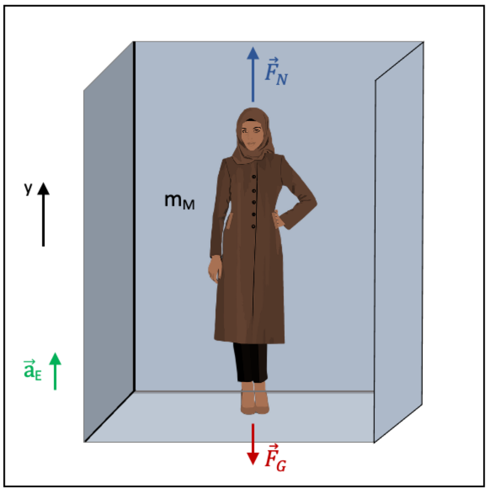


*Figure 24*

Since the scale in the elevator indicates a 15% increase of María Elena's weight, we find the magnitude of the elevator's acceleration $\vec{a}_E$ as follows (whereby the magnitude of the normal force when María Elena stands in her bathroom is equal to her weight):

$$F_N = m_M \cdot a_E + m_M \cdot g$$

$$\Leftrightarrow\ 1.15 \cdot (m_M \cdot g) = m_M \cdot a_E + m_M \cdot g$$

$$\Leftrightarrow\ a_E = (1.15 - 1) \cdot g = 1.47 \text{ m/s}^2$$

Given that María Elena observes the statues moving downwards with an acceleration $\vec{a}_{S*}$ from the perspective of an accelerating reference frame, i.e., the elevator, we find the acceleration $\vec{a}_S$ of the statues with respect to someone standing on the ground (i.e., an inertial framework) in the following way:

$$\vec{a}_S = \vec{a}_{S*} + \vec{a}_E$$

$$\Leftrightarrow\ \vec{a}_S = (-0.450 - 1.47) \cdot \vec{i}_x = -1.92 \cdot \vec{i}_x \text{ m/s}^2$$

Note hereby that the vectors $\vec{a}_{S*}$ and $\vec{a}_E$ in Fig. 23 point towards the negative direction of the x-axis of the coordinate system corresponding to mass $m_1$ which explains the two minus signs.

Let us for a moment place the dodecagonal-shaped block in an inertial framework, so that we can determine the value of the mass $m_3$ of statue number 3. Since the statues are accelerating at a rate $\vec{a}_S$, applying Newton's second law to the five statues provides the following equations (note that we have kept the minus sign of the acceleration $\vec{a}_S$ still within the variable $a_S$ until the last step of our calculations):

$$\begin{cases} m_1 \cdot a_S = T_1 - m_1 \cdot g & m_4 \cdot a_S = -T_3 + T_4 + m_4 \cdot g \cdot \sin\theta \\ m_2 \cdot a_S = -T_1 + T_2 - m_2 \cdot g \cdot \sin(2\theta) & m_5 \cdot a_S = -T_4 + m_5 \cdot g \cdot \sin(2\theta) \\ m_3 \cdot a_S = -T_2 + T_3 & \end{cases}$$

Based on the equation of statue 1 and 2 and of statue 4 and 5, we can write the following expression for $T_2$ and $T_3$, respectively:

$$\begin{cases} T_2 = (m_1 + m_2) \cdot a_S + [m_1 + m_2 \cdot \sin(2\theta)] \cdot g \\ T_3 = -(m_4 + m_5) \cdot a_S + [m_4 \cdot \sin\theta + m_5 \cdot \sin(2\theta)] \cdot g \end{cases}$$

Plugging the above two expressions into the equation of statue 3 allows us to calculate the mass $m_3$ of statue number 3:

$$m_3 \cdot a_S = -T_2 + T_3$$

$$\Leftrightarrow \; m_3 \cdot a_S = -\left[(m_1 + m_2) \cdot a_S + [m_1 + m_2 \cdot \sin(2\theta)] \cdot g\right] + \left[-(m_4 + m_5) \cdot a_S + [m_4 \cdot \sin\theta + m_5 \cdot \sin(2\theta)] \cdot g\right]$$

$$\Leftrightarrow \; m_3 = \frac{\left[-(m_1 + m_2 + m_4 + m_5) \cdot a_S + [-m_1 - m_2 \cdot \sin(2\theta) + m_4 \cdot \sin\theta + m_5 \cdot \sin(2\theta)] \cdot g\right]}{a_S}$$

$$= \frac{[-(4.60+3.30+2.80+3.40)\cdot(-1.92)+[-4.60-3.30\cdot\sin(60.0°)+2.80\cdot\sin(30.0°)+3.40\cdot\sin(60.0°)]\cdot 9.81]}{-1.92}$$

$$= 1.80 \text{ kg}$$

With a mass of $m_3 = 1.80$ kg for statue number 3, María Elena will see the statues shift to the left with an acceleration of $\vec{a}_{S*} = \vec{a}_S - \vec{a}_E = -1.92 \cdot \vec{i}_x - (-1.47 \cdot \vec{i}_x) = -0.450 \cdot \vec{i}_x$ m/s$^2$ with respect to her moving framework.

# Exercise 20

## ***Problem Statement***

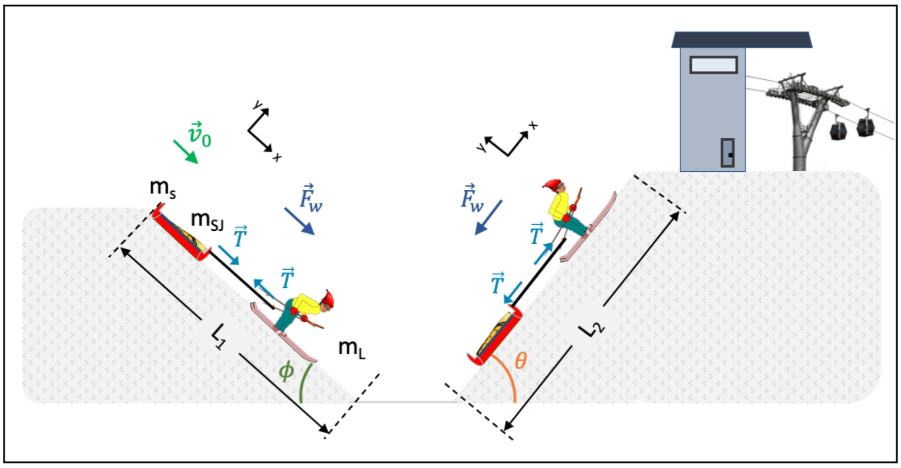


***Figure 25***

Lovisa ($m_L$ = 58.6 kg) is a seasoned rescue professional in the ski area of Åre, Sweden, and with her rescue stretcher ($m_s$ = 12.2 kg), which is attached to Lovisa's rescue gear with the help of two metal rods, she just picked up Seo Joon ($m_{SJ}$ = 85.1 kg), a South Korean tourist who injured his hip, and is on her way back to the nearest cable car station. Lovisa is standing at the top of a hill and must now gain sufficient speed to reach the station, which is located on top of the next hill. Because she gave away her ski poles to another person in need of rescue, Lovisa is hoping that an initial speed of $v_0 = 4.30$ m/s, gravity, and a constant wind in her back ($F_w = 38.5$ N) on the way down are able to get her to the top of the next hill. Assume that the wind only impacts Lovisa, since Seo Joon is lying close to the ground. (1) If the first slope is $L_1 = 84.0$ m long with an incline of $\phi = 17.8°$ and given that the second hill is 2.00 m higher with a 10.0 m shorter slope and that the wind has turned 180° from the moment she starts moving up the second hill, will Lovisa make it to the cable car station? (2) When Lovisa eventually comes to a halt, what is the tension force in the metal rod? (3) In case that Lovisa does not reach the station, what force should she exert upon her skis in order to accelerate up the hill at $a_u = 1.25$ m/s$^2$?

## ***Solution***

(1) In order to determine the average acceleration $a_1$ with which Lovisa coasts down the hill, we apply Newton's second law to Lovisa and the subsystem "Seo Joon plus stretcher". Keep in mind that the connection system between Seo Joon and Lovisa is not a rope but two metal rods, which means that, since the subsystem "Seo Joon plus stretcher" has a greater mass than Lovisa and therefore exhibits a greater inertia, the rods are pulling on the subsystem to the right when Lovisa is going downhill.

| Lovisa (downhill) | Seo Joon plus stretcher (downhill) |
|---|---|
| $m_L \cdot a_1 = -T + F_w + m_L \cdot g \cdot \sin\phi$ | $(m_{SJ} + m_s) \cdot a_1 = T + (m_{SJ} + m_s) \cdot g \cdot \sin\phi$ |

The value of the acceleration $a_1$ is found when replacing the tension force T in Lovisa's equation by the expression for T obtained from the equation corresponding to the subsystem "Seo Joon plus stretcher":

$$m_L \cdot a_1 = [-(m_{SJ} + m_s) \cdot a_1 + (m_{SJ} + m_s) \cdot g \cdot \sin\phi] + F_w + m_L \cdot g \cdot \sin\phi$$

$$\Leftrightarrow\ a_1 = \frac{F_w + (m_L + m_{SJ} + m_s) \cdot g \cdot \sin\phi}{m_L + m_{SJ} + m_s}$$

$$= \frac{38.5 + (58.6 + 85.1 + 12.2) \cdot 9.81 \cdot \sin(17.8°)}{58.6 + 85.1 + 12.2}$$

$$= 3.25\ \mathrm{m/s^2}$$

The velocity $v_b$ at which Lovisa reaches the bottom of the valley is then found as follows:

$$v_b^2 - v_0^2 = 2 \cdot a_1 \cdot L_1$$

$$\Leftrightarrow\ v_b = \sqrt{v_0^2 + 2 \cdot a_1 \cdot L_1}$$

$$= \sqrt{4.30^2 + 2 \cdot 3.25 \cdot 84.0}$$

$$= 23.7\ \mathrm{m/s}$$

Since the height $h_1$ of the first hill is measured as $h_1 = L_1 \cdot \sin\phi = 84.0 \cdot \sin(17.8°) = 25.7$ m and the angle $\theta$ of the second incline is equal to $\theta = \sin^{-1}\left(\frac{h_1+2.00}{L_1-10.0}\right) = \sin^{-1}\left(\frac{27.7}{74.0}\right) = 22.0°$, we can now write Newton's second law for Lovisa and the subsystem "Seo Joon plus stretcher" going up the second hill. Bear in mind that due to the greater inertia of the subsystem "Seo Joon plus stretcher", it takes more time to slow down with respect to Lovisa, which translates into the fact that the subsystem is pushing on the rods, which, in turn, are pushing back on the subsystem (as per Newton's third law). This explains why the tension forces in the uphill situation are pointing in the opposite direction compared to the downhill situation.

Lovisa (uphill)

$$m_L \cdot a_2 = T - F_w - m_L \cdot g \cdot \sin\theta$$

Seo Joon plus stretcher (uphill)

$$(m_{SJ} + m_s) \cdot a_2 = -T - (m_{SJ} + m_s) \cdot g \cdot \sin\theta$$

The acceleration $a_2$ is calculated by replacing T in Lovisa's equation by the expression for T obtained from the equation of the subsystem "Seo Joon plus stretcher":

$$m_L \cdot a_2 = -[(m_{SJ} + m_s) \cdot a_2 + (m_{SJ} + m_s) \cdot g \cdot \sin\theta] - F_w - m_L \cdot g \cdot \sin\theta$$

$$\Leftrightarrow\ a_2 = \frac{-[F_w + (m_L + m_{SJ} + m_s) \cdot g \cdot \sin\theta]}{m_L + m_{SJ} + m_s}$$

$$= \frac{-[38.5 + (58.6 + 85.1 + 12.2) \cdot 9.81 \cdot \sin(22.0°)]}{58.6 + 85.1 + 12.2}$$

$$= -3.92 \text{ m/s}^2$$

Under these conditions, Lovisa is able to travel the following distance $d$ on the second incline (whereby she comes to a halt when $v_f = 0$ m/s):

$$v_f^2 - v_b^2 = 2 \cdot a_2 \cdot d$$

$$\Leftrightarrow d = \frac{v_f^2 - v_b^2}{2 \cdot a_2} = \frac{0^2 - 23.7^2}{2 \cdot (-3.92)} = 72.0 \text{ m}$$

In other words, Lovisa is $L_2 - d = 74.0 - 72.0 = 2.02$ m short of reaching the cable car station.

(2) At the moment when Lovisa comes to a halt, although $v_f = 0$ m/s, the net force $m_L \cdot a_2$ in Lovisa's above equation related to Newton's second law is not zero, so that the magnitude of the tension force $\vec{T}$ is calculated as follows:

$$m_L \cdot a_2 = T - F_w - m_L \cdot g \cdot \sin\theta$$

$$\Leftrightarrow T = F_w + m_L \cdot (a_2 + g \cdot \sin\theta)$$

$$= 38.5 + 58.6 \cdot (-3.92 + 9.81 \cdot \sin(22.0°))$$

$$= 24.0 \text{ N}$$

Given that two metal rods connect the stretcher with Lovisa, the magnitude of the tension force $\vec{T}_r$ in each rod then becomes $T_r = \frac{T}{2} = \frac{24.0}{2} = 12.0$ N.

(3) If Lovisa exerts a force $-\vec{F}_s$ on her skis, then her skis are exerting a force $\vec{F}_s$ upon Lovisa, so that we can write the following equations when applying Newton's second law in the x-direction (keep in mind that the tension force $\vec{T}$ has switched direction in this situation, as it takes more effort for the subsystem "Seo Joon plus stretcher" to start moving due to its greater inertia, so that the rods end up pulling on both the subsystem and Lovisa):

| Lovisa | Seo Joon plus stretcher |
|---|---|
| $m_L \cdot a_u = F_s - T - F_w - m_L \cdot g \cdot \sin\theta$ | $(m_{SJ} + m_s) \cdot a_u = T - (m_{SJ} + m_s) \cdot g \cdot \sin\theta$ |

Based on the equation for the subsystem "Seo Joon plus stretcher", the magnitude of the tension force $\vec{T}$ is equal to:

$$T = (m_{SJ} + m_s) \cdot (a_u + g \cdot \sin\theta) = (85.1 + 12.2) \cdot [1.25 + 9.81 \cdot \sin(22.0°)] = 479 \text{ N}$$

Based on the equation for Lovisa, we find the magnitude of the force $\vec{F}_s$ as follows:

$$m_L \cdot a_u = F_s - T - F_w - m_L \cdot g \cdot \sin\theta$$

$$\Leftrightarrow \; F_s = T + F_w + m_L \cdot (a_u + g \cdot \sin\theta)$$

$$= 479 + 38.5 + 58.6 \cdot [1.25 + 9.81 \cdot \sin(22.0°)]$$

$$= 805 \text{ N}$$

The force that Lovisa exerts upon her skis is then equal to $\vec{F}_s = -805 \cdot \vec{i}_x$ N.